(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

COURSE CODE: MAT 421

COURSE TITLE: PDE I

DATE: 07/12/18 **TIME:** 11.30-1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the term partial differential equation (1mark)

b) What is meant by the term order of a partial differential equation (1mark)

c) Let $u = u(x, y)$. Find the general solution to $u_x = 2x, u(0, y) = lny$ (3marks)

d) Using the initial conditions $u(x, t) = t$ and $\frac{\partial u}{\partial x}(0, t) = e^{-t}$, find the exact solution to the pde (5mks)

$$\frac{\partial^2 u}{\partial x^2} = 6xe^{-t}$$

e) Determine $c_1$and c_2 so that $y(x) = c_1e^{2x} + c_2e^{2x} + 2sinx$ will satisfy the conditions $y(0) = 0$ and $y'(0) = 1$ (5 marks)

f) Verify that $u(x, t) = (5x - 6\boldsymbol{x}^5 + \boldsymbol{x}^9)\boldsymbol{t}^6$ satisfies the $\boldsymbol{pde}$ (15 marks)

$$x^3t^2u_{xtt} - 9x^2t^2u_{tt} = tu_{xxt} + 4u_{xx}$$

QUESTION TWO (20 MARKS)

Consider the 1-D heat equation of the form

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

This is a function defined on the spatial domain $\mathbf{0 \leq x \leq 3}$ and $\boldsymbol{t} > \mathbf{0}$. Using separation of variables, find the general solution to the $\boldsymbol{e}$, discussing the three distinct cases affecting the general solution and commending on the various outcomes of the solution (20marks)

QUESTION THREE (20 MARKS)

a) Proof that D' Alembert's formular $u(x, t) = u_1(x, t) + u_2(x, t)$ where $u_1(x, t) = \phi(x + ct)$ and $u_2(x, t) = \psi(x - ct)$ and $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are arbitrary functions, is a solution to the 1-D wave equation $\boldsymbol{u_{tt}(x, t) = c^2u_{xx}}$ (15marks)

b) Solve the wave equation below by D' Alambert's formular;

$u_{tt}(x,t) = u_{xx}$, $(x,0) = sin\ 5x$, $u_t(x,0) = \frac{1}{5}cos\ x$ (5marks)

QUESTION FOUR (20 MARKS)

Let $\boldsymbol{u(x,t)}$ represent the temperature of a very thin rod of length $\boldsymbol{\pi}$**,** which is placed on the interval $\{\boldsymbol{x/0 \le x \le \pi}\}$, at position $\boldsymbol{x}$ and time $\boldsymbol{t}$. The $\boldsymbol{pde}$ which governs the heat distribution is given by

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$$

where u, x, t and k are given proper units. Using the B.C $u(0,t) = u(\pi,t) = 0$ for $t \ge 0$ and the I.C $u(x,0) = 2sin4x - 11sin7x, for\ 0 \le x \le \pi$, find a solution $\boldsymbol{u(x,t)}$.

(20marks)

QUESTION FIVE (20 MARKS)

Let $\boldsymbol{u(x,t)}$ represent the vertical displacement of a string of length $\boldsymbol{\pi}$**,** which is placed on the interval $\{\boldsymbol{x/0 \le x \le \pi}\}$, at position $\boldsymbol{x}$ and time $\boldsymbol{t}$. The $\boldsymbol{pde}$ which governs the displacement is given by

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$$

where $\boldsymbol{u, x, t}$ and $\boldsymbol{k}$ are given proper units. Using the B.C $\boldsymbol{u(0,t) = u(\pi,t) = 0}$ **for** $\boldsymbol{t \ge 0}$ and the I.C $\boldsymbol{u(x,0) = 5sin3x - 6sin8x,\ for\ 0 \le x \le \pi}$, and initial velocity $\boldsymbol{u_t(x,0) = 0}$ for $\boldsymbol{0 \le x \le \pi}$, solve the equation for $\boldsymbol{u(x,t)}$. (20marks)