(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: MAT 421

COURSE TITLE: PDE I

DATE: 15/12/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 Mks)

(a) Define the terms:
(i)linear, (ii) semi-linear, and (iii) quasi-linear partial differential equation of first order (3Mks)

(b) Obtain the solution of the first order linear partial differential equation
$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} + xu = 0$ satisfying the condition $u = y$ when $x^2 + 2y^2 = 4$ (8Mks)

(c) If $Z = Z(x, y)$ is a function of x and y, eliminate the arbitrary function f from the equation (i) $Z = x + f(x^2 - y^2)$ (2Mks)

(ii) $Z = e^{\frac{y}{x}} f(xy)$ (3Mks)

(d) Distinguish Pfaffian differential form and a Pfaffian differential equation (2Mks)

(e) Obtain the integral curves of the equations $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{(x+y)z}$ (6Mks)

(f) Use the Jacobi's method to find the complete integral of the first order partial differential equation $p^2 - q^2 = z$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ (6Mks)

QUESTION TWO (20 Mks)

Given a surface by the parametric equations

$x = u + v$

$y = u - v$

$z = 4uv$

Where u and $v$are real values

(a) Show that the above equation represents a surface of jacobians. (10Mks)

(b) Find (a) above by eliminating u and v (10Mks)

QUESTION THREE(20 Mks)

(a) Verify that the Pfaffian differential equation
$yzdx + (x^2y - xz)dy + (x^2z - xy)dz = 0$ is integrable. (8Mks)

(b) Find the general solution and obtain the particular solution satisfying the condition: $z = y$ when $x = 1$ (5Mks)

(c) Use Chapit's method of solving non-linear first order partial differential equations to find the complete integral of the equation $Z^2 = pqxy$ (7Mks)

QUESTION FOUR (20 Mks)

(a) Using the fact that coefficients of the differential equation $(y^2+z^2)dx+xydy+xzdz=0$ are homogeneous functions in x, y and z of the same degree obtain the general solution. (6Mks)

(b) Show that the first order partial differential equation $xp-yq=x, x^2p+q=xz$, are compatible. (7Mk)

(c) Hence obtain the general solution of the given simultaneous equations and find the solution passing through the point $(1,1,3)$ (7Mks)

QUESTION FIVE (20 Mks)

(a) Consider the Lagrange's partial differential equation $x(y-z)p+y(z-x)q=z(x-y)dz$ where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ write the corresponding Lagrange's auxiliary equations and hence find its integral surface that passes through the curve $z=1, y=x$. (13Mks)

(b) Find the equation of the tangent plane to the hyperboloid $4x^2-9y^2-9z^2-36=0$ at the point $(3\sqrt{3},2,2)$ (7Mks)