(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 314

COURSE TITLE: ODE

DATE: 28/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) The slope at any point of a curve is $2x+3y$. If the curve passes through the origin, determine its equation. (5 Marks)

b) Given that $y = x$ is a solution of $x\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ at $x \neq 0$. Find the general solution of $x\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x$ (4 Marks)

c) Prove that $L[Sin(at)] = \frac{a}{s^2 - a^2}$ (4 Marks)

d) By use of separable variables, solve first order differential equation $(1-x)dy + (1-y)dx = 0$ (4 Marks)

e) Solve a homogeneous equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$. (4 Marks)

f) Solve a differential equation below which is reducible homogeneous form.

$$\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$$ (6 Marks)

g) Define the following terms as used in ordinary differential equations. (3 Marks)
   (i) Operator
   (ii) Partial Differential Equation
   (iii) Linear Differential Equation.

QUESTION TWO (20 MARKS)

a) Solve $(D^3 - D^2 - 6D)y = x^2 + 1$ (7 Marks)

b) Apply partial differential method to solve $z^2(p^2x^2 + x^2) = 1$ (8 Marks)

c) Evaluate the functional $I = \int_0^1 \left[y^2 + \left(\frac{dy}{dx}\right)^2 \right] dx$ by calculus variations along the paths (i) $y = x^2$ (ii) $y = (e^x - 1)(e - 1)$ (5 Marks)

QUESTION THREE (20 MARKS)

a) Find the Laplace transforms of the following functions.
   (i) $f(x) = e^{ax}$ (5 Marks)
   (ii) $f(x) = Sinh(ax)$ (7 Marks)

b) Solve the function $y = yp^2 + 2px$ given that the function is solvable for y only and $y = f(x,p), f(x,p,c) = 0, f(x,y,p) = 0, p = \frac{dy}{dx} = Q\left(x, p\frac{dp}{dx}\right)$ (5 Marks)

c) How long does it take for a given amount of money to double at 6% interest rate per annum compounded
   (i) Annually. (1 Marks)
   (ii) Continuously. (2 Marks)

QUESTION FOUR (20 MARKS)

a) Solve a homogeneous differential equation $(D^3+1)y = Cos2x$ when $Q(x) = bSin(ax)...or....bCos(nx)$ (10 Marks)

b) Using Multiplier method, solve $\frac{dx}{x(y^2-z^2)} = \frac{dy}{-y(z^2+x^2)} = \frac{dz}{z(x^2+y^2)}$ (10 Marks)

QUESTION FIVE (20 MARKS)

a) Compound Z is formed when two chemicals X and Y are combined. The resulting reaction between the two chemicals is such that each gram of X, $4g$ of Y is used. It is observed that $30grams$ of compound Z is formed in $10Minutes$. Determine the amount of Z at any time if the rate of reaction is proportional to the amount of X and Y remaining when initially there were $50grams$ of X and $32grams$ of Y. How much compound Z is present after $15Minutes$. Interpret the solution as $t \to \infty$ (10 Marks)

b) Find the Inverse transforms of $\frac{5s+2}{(s-2)^2+13}$ (4 Marks)

c) In each of the following types of equation write two examples. (6 Marks)
   (i) First Order Differential Equations
   (ii) Second Degree Differential Equations
   (iii) Ordinary Differential Equations