(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: MAT 314

COURSE TITLE: ODE

DATE: 18/10/2018   **TIME:** 11.30 AM- 1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms as used in ordinary differential equations.   (3 Marks)
   (i)   Operator
   (ii)   Partial Differential Equation
   (iii) Linear Differential Equation.

b) The slope at any point of a curve is $3x + 2y$. If the curve passes through the origin, determine its equation.   (5 Marks)

c) Given that $y = x$ is a solution of $x\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} - y = 0$ at $x \neq 0$. Find the general solution

   of $x\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} - y = x$ (4 Marks)

d) Solve a homogeneous equation $xdy - ydx = \sqrt{x^2 + y^2}\,dx$.   (4 Marks)

e) Prove that $L[Sin(at)] = \dfrac{a}{s^2 - a^2}$ (4 Marks)

f) By use of separable variables, solve first order differential
   equation $(1 - x)dy + (1 - y)dx = 0$ (4 Marks)

g) Solve a differential equation below which is reducible homogeneous form .
   $\dfrac{dy}{dx} = \dfrac{x - y + 3}{2x - 2y + 5}$ (6 Marks)

QUESTION TWO (20 MARKS)

a) Compound Z is formed when two chemicals X and Y are combined. The resulting reaction between the two chemicals is such that each gram of X, $6g$ of Y is used. It is observed that $40\,grams$ of compound Z is formed in $15\,Minutes$. Determine the amount of Z at any time if the rate of reaction is proportional to the amount of X and Y remaining when initially there were $60\,grams$ of X and $42\,grams$ of Y. How much compound Z is present after $15\,Minutes$. Interpret the solution as $t \to \infty$ (10 Marks)

b) Find the Inverse transforms of $\dfrac{5s + 2}{(s - 2)^2 + 13}$ (4 Marks)

c) In each of the following types of equation write two examples.   (6 Marks)
   (i)   First Order Differential Equations
   (ii)   Second Degree Differential Equations
   (iii) Ordinary Differential Equations

QUESTION THREE (20 MARKS)

a) Find the Laplace transforms of the following functions.

   (i) $f(x) = e^{ax}$ (5 Marks)

   (ii) $f(x) = Sinh(ax)$ (7 Marks)

b) Solve the function $y = yp^2 + 2px$ given that the function is solvable for y only and

$$y = f(x, p), f(x, p, c) = 0, f(x, y, p) = 0, p = \frac{dy}{dx} = Q\left(x, p\frac{dp}{dx}\right)$$ (5 Marks)

c) How long does it take for a given amount of money to double at 6% interest rate per annum compounded

   (i)   Annually.   (1 Marks)

   (ii)   Continuously.   (2 Marks)

QUESTION FOUR (20 MARKS)

a) Solve a homogeneous differential equation $(D^3 + 1)y = \sin 2x$ when

$Q(x) = bSin(ax)...or....bCos(nx)$ (10 Marks)

b) Using Multiplier method, solve $\dfrac{dx}{x(y^2 - z^2)} = \dfrac{dy}{-y(z^2 + x^2)} = \dfrac{dz}{z(x^2 + y^2)}$ (10 Marks)

QUESTION FIVE (20 MARKS)

a) Solve $(D^3 - D^2 - 6D)y = x^2 + 1$ (7 Marks)

b) Apply partial differential method to solve $z^2(p^2x^2 + x^2) = 1$ (8 Marks)

c) Evaluate the functional $I = \int_0^1 \left[y^2 + \left(\frac{dy}{dx}\right)^2 \right] dx$ by calculus variations along the paths (i)

$y = x^2$ (ii) $y = (e^x - 1)(e - 1)$ (5 Marks)