(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 314**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATIONS**

DATE: 15/01/18 **TIME:** 9.00 A.M-11.00 A.M.

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms as used in ordinary differential equations. (4 Marks)
   i) Differential Equation
   ii) Order of Differential Equation
   iii) Linear Differential Equation
   iv) Integrating Factor.

b) Solve $(1-x)dy+(1-y)dx=0$ (4 Marks)

c) Solve differential equation $\left(x+2y^{3}\right)\frac{dy}{dx}=y$ (3 Marks)

d) State two main classifications of differential equations. (2 Marks)

e) How long does it take for a given amount of money to double at 6% interest rate per annum compounded
   (i) Annually. (2 Marks)
   (ii) Continuously. (2 Marks)

f) Solve $\left(D^{2}+2D+1\right)y=2x+x^{2}$ when $Q(x)=bx^{k}$ and $P(D)=D-a_{0}, a_{0}\neq 0$ (5 Marks)

g) Solve the Legendre Linear equation below

$$(1+x)^{2}\frac{d^{2}y}{dx^{2}}+(1+x)\frac{dy}{dx}+y=2Sin[\log(1+x)]$$ (4 Marks)

h) Prove that $L(Cosh(at))=\frac{s}{s^{2}-a^{2}}$ (4 Marks)

QUESTION TWO (20 MARKS)

a) Solve a differential equation reducible to homogeneous form below
   $(2x+y-3)dy=(x+2y-3)dx$ (8 Marks)

b) Solve the homogeneous differential equation $\left(x^{2}-y^{2}\right)dx+2xydy=0$ (6 Marks)

c) Find the differential equation corresponding to the curve $y=C(x-C)^{2}$ where C is arbitrary constant. (6 Marks)

QUESTION THREE (20 MARKS)

a) Solve homogeneous differential equation $x\frac{dy}{dx}=y(\log y-\log x+1)$ (4 Marks)

b) Apply partial differential equation method to solve $y\frac{\partial^{2}xy}{\partial x\partial y}+\frac{\partial z}{\partial x}=4xy$ (8 Marks)

c) By use of calculus of variation evaluate the functional $I=\int_{0}^{1}\left(y+,x\frac{\partial y}{\partial x}\right)dx$ along the path
   (i) $y=x$ (ii) $y=2x^{2}$ and (iii) $y=e^{x}$ from $(0,0)$ to $(1,1)$ (6 Marks)

d) Highlight two importance of Laplace transforms concepts. (2 Marks)

QUESTION FOUR (20 MARKS)

a) Use first shifting property of Laplace transform to find the Laplace transform of $Sin2xSin3x$ hence show that

$$L[xSin(ax)] = \frac{2as}{(s^2+a^2)^2}, \quad L[xCos(ax)] = \frac{s^2-a^2}{(s^2+a^2)^2}$$ (10 Marks)

b) $2g$ of substance Y combines with $1g$ of substance X to form $3g$ of Z. When $100g$ of Y is thoroughly mixed with $50g$ of X, it is found that in $100Minutes$ $50g$ of Z has been formed. How many grams of Z can be formed in $20Minutes$? And how long does it take to form $60g$ of Z. (10 Marks)

QUESTION FIVE (20 MARKS)

a) Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{y^2x}$ by grouping method. (6 Marks)

b) Find the inverse transform of $\frac{5s+3}{(s-1)(s^2+2s+s)}$ (6 Marks)

c) Find the differential Equation that describes the family of circles passing through the origin. (8 Marks)