(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **MAT 405**

COURSE TITLE: **MEASURE THEORY**

DATE: 13/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 COMPULSORY (30 MKS)

a). Define area as the measure of the interval on $\mathbb{R}$ given that $a < x < b$ and $c < x < d$ with $a, b, c, d \in \mathbb{R}$ **(4 mks)**

b) Prove that $\mu^*(\{x\}) = 0$ **(5 mks)**

c) State two examples of $\sigma - algebra$ **(2 mks)**

d) Define a measurable space **(4 mks)**

e) Show that the outer measure is count ably sub additive **(15 mks)**

QUESTION 2 : (20 mks)

a) Let $A_1, A_2, \ldots\ldots\ldots, A_n$ be a pairwise disjoint collection of members of lebesque measurable sets. Then for any $X \subseteq \mathbb{R}$ prove that $\mu^*(X \cap \bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} \mu^*(X \cap A_i)$ **(10 mks)**

b) Let $(X, \mathfrak{X}, \mu)$ be a measure space $f, g \in M^+(X, \mathfrak{X})$and c a non-negative real constant, show that $\int (f + g)\, d\mu = \int f\, d\mu + \int g\, d\mu$ **(10 mks)**

QUESTION 3 : (20 mks)

a) Show that a non- degenerate interval of $\mathbb{R}$ is uncountable **(8 mks)**

b) State the three types of measures of the intervals over $\mathbb{R}$and illustrate them on separate diagrams **(6mks)**

c) Let $(X, \mathfrak{X})$ be a measurable space and $f : X \rightarrow \mathbb{R}$, $g : X \rightarrow \mathbb{R}$ functions. Prove that $g + f$ is $\mathfrak{X} -$ measurable function for all $x \in X$. **(6 mks)**

QUESTION 4 : (20 mks)

a) Given that a borel set $B(\mathbb{R})$ is a $\sigma - algebra$ of subset of $\mathbb{R}$ and by definition of topological space state the structure of Borel set if $a, b \in \mathbb{R}$ **(12mks)**

b)Let $\{\mathfrak{X}_\alpha : \alpha \in \mathbb{N}\}$ be a family of $\sigma - algebra$ of subsets of X . Prove that the set $\cap \mathfrak{X}_\alpha$ $such\ that\ \alpha\ is\ from\ the\ index\ set$ is also a $\sigma - algebra$ of subset of X.

(8 mks)

QUESTION 5: (20 mks)

a) Prove the translation invariance property of the outer measure **(5 maks)**

b) If $B, A \subseteq \mathbb{R}$ and that $A \subseteq B$, with the$\mu^*(A) = 0$, show that

$$\mu^*(B) \leq \mu^*(A \cup B)$$ **(5 mks)**

c) c) If $E \subseteq \mathbb{R}$ and $\mu^*(E) = 0$,then prove that $E \in \mathcal{M}$ and also if $F \subset E$ then $F \in \mathcal{M}$ (lebesque measurable) **(10 mks)**