(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

(MATHEMATICS)

COURSE CODE: **MAT 405**

COURSE TITLE: **MEASURE THEORY**

DATE: 03/10/18 **TIME:** **3 PM -5 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (COMPULSORY)(30 marks)

a) Define the term measurable space , hence name any two examples of measurable spaces **(5mks)**

b) Let $A, B \subseteq \mathbb{R}$ and $\mu*(A) = 0$ show that

$\mu*(A \cup B) = \mu*(B)$ **(5mks)**

c) State any three properties that are satisfied by the outer measure $\mu*$ **(6mks)**

d) State without proof lebesque monotone convergent theorem. **(4mks)**

e) Prove that the outer measure is countably sub - additive.

i.e.

$$\mu^*(\cup_{n=1}^{\infty} E_n) \leq \sum_{n=1}^{\infty} \mu^*(E_n) \;\; \forall\; n = 1,2,3,4, \ldots\ldots\ldots \infty$$

(10 marks)

QUESTION TWO (20 marks)

Let $(X, \mathbb{X}, \mu)$ be a measure space and (f_n) a monotone increasing sequence of elements of $m^+(X, \mathbb{X})$ converging to f pointwise on X . Then prove that

$\lim_{n\to\infty} \int f_n \, d\mu = \int f d\mu$ **(20 marks)**

QUESTION THREE(20 marks)

a) Let $(X, \mathbb{X}, \mu)$ be a measurable space and (f_n) a sequence of elements from $m^+(X, \mathbb{X})$ then prove that ,

$\int \underline{\lim}_{n\to\infty} f_n \, d\mu \leq \underline{\lim}_{n\to\infty} \int f_n \, d\mu$ **(16 marks)**

b) Define an algebra **(4 marks)**

QUESTION FOUR (20 marks)

a) Let $(X, \mathfrak{X}, \mu)$ be a measure space $f, g \in M^+(X, \mathfrak{X})$ and c a non-negative real constant, show that $\int (f+g)\, d\mu = \int f\, d\mu + \int g\, d\mu$

(10 mks)

b) Let X *and* Y be none – empty sets and ¥ be a $\sigma - Algebra$ of subsets of Y.

Let $f : X \to \mathbb{R}$ be a function and $\text{X} = \{f^{-1}(E) : E \in ¥\}$.

Then show that X is a $\sigma - Algebra$ of subsets of X. **(10 mks)**

QUESTION FIVE(20 marks)

a) Show that the measure is additive

i.e $\mu(\bigcup_{i=1}^{n} E_i = \sum_{i=1}^{n} \mu(E_i)$ **(8mks)**

b) Let $A, B \subseteq \mathbb{R}$ with $\mu*(A) < \infty, \mu*(B) < \infty$

Prove that $|\mu^*(B) - \mu^*(A)| \leq \mu^*(A\Delta B)$ **(6 mks)**

c) If $E \subseteq \mathbb{R}$ and $\mu^*(E) = 0$ then prove that $E \in \mathcal{M}$ **(6 mks)**