(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR EDUCATION AND BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAT 405

COURSE TITLE: MEASURE THEORY

DATE: 22/12/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 : (30MKS) COMPULSORY

a) Given a non-void interval **I** of $\mathbb{R}$ with end points a, b $\in \mathbb{R}$ such that $a < b$. State any three nature of intervals which are not infinity. Hence determine the length of the interval if $a = b$ **(6mks)**

b) Describe the three types of measure over $\mathbb{R}$ with the help of the diagram **(6mks)**

c) Given non-empty set $\emptyset$ and that the set $\emptyset \in \mathcal{M}$. (Lebesque measurable) Then prove that $\emptyset^c \in \mathcal{M}$ **(5 mks)**

d) Let A, B be subsets of $\mathbb{R}$ and $\mu^*(B) = 0$ Prove that
$$\mu^*(A \cup B) = \mu^*(A)$$
(5 mks)

e) Let Let $(X, \mathfrak{X}, \mu)$ be a measurable space $A, B \in \mathfrak{X}$ and $A \subseteq B$ and $f, g \in M^+(X, \mathfrak{X})$ and $f \leq g$ Then show that $\int f \, d\mu \leq \int g \, d\mu$ **(4 mks)**

f) Show that $\mu^*(\{x\}) = 0$ where $x \in \mathbb{R}$ **(4 mks)**

QUESTION 2 :(20 mks)

a) Let $A_{1,}\ A_2, \dots \dots \dots, A_n$ be subsets of $\mathbb{R}$. Prove that

$\mu^*\left(\bigcup_{j=1}^{n} A_j\right) \leq \sum_{j=1}^{n} \mu^*(A_j)$ i.e the outer measure is finitely sub-additive

(10 mks)

b) Let $(X, \mathfrak{X}, \mu)$ be a measurable space $f, g \in M^+(X, \mathfrak{X})$and c a non-negative real constant, show that $\int cf \, d\mu = c \int f \, d\mu$

(10 mks)

QUESTION 3 : (20 mks)

a) Define the term Lebesque measurable set **(4 mks)**

b) If $E \subseteq \mathbb{R}$ and $\mu^*(E) = 0$,

i) then prove that $E \in \mathcal{M}$

ii) further if $F \subset E$ then prove that $F \in \mathcal{M}$ **(10 mks)**

c) If $B, A \subseteq \mathbb{R}$ and that $B \supseteq A$, show that

$$\mu^*(B) \geq \mu^*(A)$$

(6 mks)

QUESTION 4 : (20 mks)

a) Let E_1 , E_2 , E_3 , E_4 ... ... E_2 be a pairwise o disjoint collection of members of μ^*. Then for any $X \subseteq \mathbb{R}$ prove that

$$\mu^*(X \cap \cup_{i=1}^{n} E_i) = \sum_{i=1}^{n} \mu^*(X \cap E_i)$$

(10 mks)

b) Let $(X, \mathfrak{X})$ be a measurable space and $f: X \to \mathbb{R}^*$ function be $\mathfrak{X}-$ measurable Prove that $cf(x)$ is $\mathfrak{X}-$ measurable function for all $x \in X$ and $c \in \mathbb{R}$

(10mks)

QUESTION 5 : (20 mks)

For the interval **I** of $\mathbb{R}$ prove that the outer measure of the interval is equal to the length of the interval i.e $\mu^*(I) = l(I)$

(20 mks)