(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR SCIENCE

COURSE CODE: **MAT 403**

COURSE TITLE: **COMPLEX ANALYSIS II**

DATE: 08/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

UNIT CODE: MAT 403 **UNIT TITLE**: COMPLEX ANALYSIS II SPECIAL EXAM

INSTRUCTIONS: Answer question one and any other two

QUESTION ONE (Compulsory)

a) Show that $\int_{-\infty}^{\infty} \frac{z^2+3}{(z^2+1)(z^2+4)} dz = \frac{5}{6}\pi$ (10 marks)

b) Determine the singularities of the following functions hence find their Laurent series

i) $f(z) = (z-3)\sin\frac{1}{z+2}$ (5 marks)

ii) $f(z) = \frac{e^{2z}}{(z-1)^3}$ (5 marks)

c) Define a Harmonic function and hence show that the function $\emptyset = x^3 - 3xy^2 + 2y$ can be a real part of analytic function. Find the imaginary part of the analytic function.

(10 marks)

QUESTION TWO

a) State and prove the Residue theorem. (5 marks)

b) Show that $\oint_c \frac{\sin z}{z^4} dz = -\frac{\pi}{3} i$, where $c :\ |z| = 1$,described in a positive direction.

(5 marks)

c) Let $f(z)$ be analytic inside and on a simple closed curve C except at a pole a of order m inside C . Prove that the residue of $f(z)$ at a is given by

$a_{-1} = \lim_{z\to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}$. Hence find the residue of the function

$f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$. (10 marks)

QUESTION THREE

a) Find $I = \int_0^{2\pi} \frac{\cos 2\theta d\theta}{5-4\sin\theta}$ (10 marks)

b) Compute the integral $\oint \frac{5z-2}{z(z-2)}dz$ around a circle radius $r = 3$ centered at the origin.

(5 marks)

c) Discuss the singularity of the following function: $f(z) = \frac{z\cos z}{(z-1)(z^2+1)^2(z^2+3z+2)}$

(5 marks)

QUESTION FOUR

a) Evaluate $\int_{-\infty}^{\infty} \frac{z^2 dz}{(z^2+1)^2(z^2+2z+2)}$ (5 marks)

b) Show that $\int_0^{2\pi} \frac{d\theta}{3-2\cos\theta+\sin\theta} = \pi$ (5 marks)

c) Consider the contour C defined by $x = y, x > 0$ and the contour C_1 defined by $x = 1, y \geq 1$. Maps these two curves using $w = \frac{1}{z}$ and verify that their angle of intersection is preserved in size and direction. (10 marks)

QUESTION FIVE

a) Find a Schwartz-Christoffel transformation that maps the upper half plane H to the inside of a triangle vertices $-1, 0$ and i . (10 marks)

b) Define the Laurent series of a function of a complex variable $f(z)$ and hence expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1 < |z| < 3$ (10 marks)