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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 402**

COURSE TITLE: TOPOLOGY II

DATE: 20/09/17 **TIME:** **11.30 AM -1.30 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1 (30 MARKS)

a) The class $\tau = \{X, \emptyset, \{a\}\{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$ is a topology on $X = \{a,b,c,d,e\}$. Show that X is disconnected. (5 marks)

b) Let F be a closed subset of a compact space X. Then F is also compact. Prove. (7 marks)

c) Let $\tau = \{X, \emptyset, \{1\}\{2,3\}\}$ be a topology on $X = \{1,2,3\}$. Show that X is a regular space. (4 marks)

d) Show that every subspace of a second countable space is second countable . (7 marks)

e) Let τ be the cofinite topology on any set X . Show that (X, τ) is separable. (7 marks)

QUESTION 2 (20 MARKS)

a) Define separated sets (2 marks)

b) Consider the following intervals on the real line $\mathbb{R}$: $A = (0,1), B = (1,2)$ and $C = [2,3)$. Show that

 i. A and B are separated. (3 marks)

 ii. B and C are not separated (3 marks)

c) Show that if A and B are non empty separated sets, then $A \cup B$ is disconnected. (6 marks)

d) Let $G \cup H$ be a disconnection of A. Show that $A \cap G$ and $A \cap H$ are separated sets. (6 marks)

QUESTION 3 (20 MARKS)

a) Prove that a topological space X is a T_1 – space if and only if every singleton subset is closed. (10 marks)

b) Consider the topology $\tau = \{X, \emptyset, \{a\}\{b\}, \{a,b\}\}$ on $X = \{a,b,c\}$. Show whether or not (X, τ) is:

 i. A normal space (5 marks)

 ii. T_1 – space (2 marks)

 iii. A regular space (3 marks)

QUESTION 4 (20 MARKS)

a) Let A be any subset of a second countable space X. If $\mathcal{G}$ is an open cover of A, then $\mathcal{G}$ is reducible to a countable cover. Prove. (6 marks)

b) The class $\tau = \{X, \emptyset, \{a\}\{c,d\}, \{a,c,d\}, \{b,c,d,e\}\}$ is a topology on $X = \{a,b,c,d,e\}$. Show that X is disconnected. (5 marks)

c) Let $B_p = \{G_1, G_2, ...\}$ be a countable local base at $p \in X$. Show that:

 i. There exists a nested local base at p. 5 marks)

 ii. If X satisfies the first axiom of countability then there exists a nested local base at every $p \in X$. (4 marks)

QUESTION 5 (20 MARKS)

a) Define compact sets. (2 marks)

b) A continuous image of a sequentially compact set is sequentially compact. Prove. (6 marks)

c) Show that if τ is a cofinite topology on any set X then (X, τ) is a compact space. (6 marks)

d) The open interval $A = (0,1)$ on the real line $\mathbb{R}$ with the usual topology is not sequentially compact. Show. (6 marks)