(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

FOURTH YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE (MATHEMATICS)

COURSE CODE: **MAT 402**

COURSE TITLE: TOPOLOGY II

DATE: 30/07/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following terms

(i). A connected space (2 mks)

(ii). A Convex set (2mks)

(iii). A second countable space (2 mks)

(iv). A Haursdorff space (2 mks)

(v). A T_1 space (2 mks)

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b) Let $X = [0,2]$, show that we cannot have a separation of X.(3 mks)

c) Show that the image of a compact set under a continuous map is compact. (4 mks)

d) Show that a subspace of a second countable space is second countable. (4 mks)

e) Let $X = \{a, b\}$. Form a topological space on X hence show that the formed topology is a topology and but notT_1.(5mks)

f) Let X be a regular space and one-point sets be closed in X. Show that if given a point $x \in X$ and a neighbourhood U of$x$, there is a neighbourhood V of $x$such that $\bar{V} \subset U$. (4mks)

QUESTION TWO (20 MARKS)

a) Define the term a compact space, hence show that space of real numbers, $\mathbb{R}$, is not compact. (4 mks)

b) Show that the set $X = \{x, y, z, k\}$ with a topology $\tau = \{\phi, \{z\}, \{y, z\}, X\}$ is not a Hausdorffspace. (3 mks)

c) Let $f: X \to Y$ be a continuous function from a topological spaces, X, to another, Y. Show that its image is connected given that$X$ is connected. (5 mks)

d) State and prove the intermediate value theorem (8 mks)

QUESTION THREE (20 MARKS)

a) (i).What is a linear continuum? (2mks)

(ii).Prove that a linear continuum L, in the order topology, as well as its intervals and rays are connected. (11mks)

b) Prove that every metrizable space is normal. (7mks)

QUESTION FOUR (20 MARKS)

a) Show that the real line $\mathbb{R}$ is locally compact. (4 mks)

b) Prove the claim that the interval $(0,1)$ on the real line is sequentially compact. (5 mks)

c) Let $X \times Y$ be a product topological space and π_1, π_2 be projections on X and Y respectively be defined as $\pi_1(x, y) = x$ and $\pi_2(x, y) = y$ for $x \in X$ and $y \in Y$. Let $A \subset X \times Y$ be square $A = \{(x, y): 0 \leq x \leq a, 0 \leq y \leq b \text{ for } a, b \in \mathbb{R}\}$. Show that A is a linear continuum. (6 mks)

d) Prove that the real line, $\mathbb{R}$, together with the usual matrix $d = |x - y|$ for $x, y \in \mathbb{R}$ is a Hausdorff space. (5 mks)

QUESTION FIVE (20 MARKS)

a) Let X be a topological space. Define a relation $x \sim y$ on X if there is a connected subspace of X containing both $x$and y. Show that $\sim$ is an equivalence relation. (5 mks)

b) Prove that every open covering of a space X with a countable basis contains a countable sub collection covering X. (5 mks)

c) Let $\{A_\alpha\}, \alpha \in I$ be a collection of connected subspaces with a common point. Show that $\bigcup_{\alpha \in I} A_\alpha$is connected. (5mks)

d) When is a function said to be uniformly continuous on a metric space (X, d) where d is a metric on X. Provide an example of uniformly continuous function and one that is not is continuous but not uniformly continuous on a given interval. (5 mks)