(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 307**

COURSE TITLE: **NUMBER THEORY**

DATE: 13/09/17 **TIME:** 8 AM -10 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

UNIT CODE: MATH 307 **UNIT TITLE**: NUMBER THEORY SPECIAL EXAM

INSTRUCTIONS: Answer question one and any other two questions

1. a) State any five simple properties of Congruences. (5 marks)

   b) Prove that if a, b and n are integers then $(a, b) = (a + nb, b)$ (5 marks)

   c) Find integers x, y which satisfy the linear diophanite equation $23x + 29y = 1$

   (5 marks)

   d) Prove the following result of Euler $641|(2^{32} + 1)$ (8 marks)

   e) State and prove the Euclid's theorem. (7 marks)

2. a) Prove that the product of n consecutive integers is divisible by $n!$ (6 marks)

   b) Prove that $n^5 - 5n^3 + 4n$ is always divisible by 120. (4 marks)

   c) Show that the greatest common divisor of any two integers a, b can be written as a linear combination of a and b (10 marks)

3. a) Prove that every year, including any leap year, has at least one Friday on 13th.

   (10 marks)

   b) How many positive integers ≤ 1260 are relatively prime to 1260? (5 marks)

   c) Show that there are infinitely many primes of the form $4n + 3$. (5 marks)

4. a) Define the term Divisibility and prove that if $7|(3x + 2)$ then

   $7|(15x^2 - 11x - 14)$ (5 marks)

   b) Find all solutions for the congruence $5x \equiv 3$ mod 7 (5 marks)

c) Define arithmetic functions f and give, with description, any five examples of arithmetic functions. (10 marks).

5. a) Show that Z_p is a ring. (5 marks)

   b) Show that Z_6 forms a complete set of Residues modulo 6. (5 marks)

   c) State and prove the Euclid's lemma of unique factorization. (5 marks)

   d) State and prove the Fermat theorems. (5 marks)