(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 307**

COURSE TITLE: **NUMBER THEORY**

DATE: 03/10/18 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

UNIT CODE: MAT 307 **UNIT TITLE**: NUMBER THEORY SPECIAL EXAM 2017

INSTRUCTIONS: Answer question one and any other two

QUESTION ONE (Compulsory)

a) Prove that $n^5 - 5n^3 + 4n$ is always divisible by 120. (5 marks)

b) Solve the congruence $3x \equiv 6 \ mod \ 12$ (5 marks)

c) Prove that 7 divides $3^{2n+1} + 2^{n+2}$ for all natural numbers n (5 marks)

d) Find x such that $x \equiv 3 \ mod \ 5$ and $x \equiv 7 \ mod \ 11$ (5 marks)

e) Show that $n^2 + 23$ is divisible by 24 for infinitely many n . (5 marks)

f) Prove that if a, b, n are positive integers, then the greatest common divisor of a and b is the same as the greatest common divisor of $a + nb$ and b . (5 marks)

Question Two

a) Prove that the set of integers is a ring? (8 marks)

b) Prove that $\log_3 4$ is irrational. (4 marks)

c) State and prove the Wilson's theorem. (8 marks)

Question Three

a) Show that every integer lies in one of the families $3k, 3k + 1,$ or $3k + 2$ where $k \in \mathbb{Z}$ (10 marks)

b) Prove that $n^4 + 4$ is prime only $n = 1$ for $n \in \mathbb{N}$ (5 marks)

c) State the Euclidean Algorithm process of finding greatest common divisor and use it to find the greatest common divisor 23 and 29. (5 marks)

Question four

a) State and prove the Euler's theorem. Use the theorem to find the last two digits of 3^{1000}

(10 marks)

b) Show that $\mathbb{Z}_3$ form a group of residues under multiplication modulo 3. (4 marks)

c) Solve the congruence $3x^2 + 3x + 2 = 0 \ (mod\ 10)$. (6 marks)

Question five

a) Find one pair of positive integers a, b such that

i) $ab(a + b)$ is not divisible by 7

ii) $(a + b)^7 - a^7 - b^7$ is divisible by 7^7 . Justify your answer. (15 marks)

b) Find all the solutions in integers to $3456x + 246y = 234$. (5 marks)