(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAT 307

COURSE TITLE: NUMBER THEORY

DATE: 11/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

UNIT CODE: MAT 307 **UNIT TITLE**: NUMBER THEORY MAIN EXAM 2017

INSTRUCTIONS: Answer question one and any other two

QUESTION ONE (Compulsory)

a) Prove that the set of integers is a ring? (8 marks)

b) Prove that $\log_3 4$ is irrational. (4 marks)

c) State the Euclidean Algorithm process of finding greatest common divisor and use it to find the greatest common divisor 23 and 29. (5 marks)

d) Show that $n^2 + 23$ is divisible by 24 for infinitely many n . (5 marks)

e) State and prove the Wilson's theorem. (8 marks)

Question Two

a) Prove that $n^5 - 5n^3 + 4n$ is always divisible by 120. (5 marks)

b) Solve the congruence $3x \equiv 6 \ mod \ 12$ (5 marks)

c) Show that every integer lies in one of the families $3k, 3k + 1,$ or $3k + 2$ where $k \in \mathbb{Z}$ (10 marks)

Question Three

a) Prove that if a, b, n are positive integers, then the greatest common divisor of a and b is the same as the greatest common divisor of $a + nb$ and b . (5 marks)

b) Find all the solutions in integers to $3456x + 246y = 234$. (5 marks)

c) Prove that 7 divides $3^{2n+1} + 2^{n+2}$ for all natural numbers n (5 marks)

d) Prove that $n^4 + 4$ is prime only $n = 1$ for $n \in \mathbb{N}$ (5 marks)

Question four

a) State and prove the Euler's theorem. Use the theorem to find the last two digits of 3^{1000}

(10 marks)

b) Show that $\mathbb{Z}_3$ form a group of residues under addition modulo 3. (4 marks)

c) Solve the congruence $3x^2 + 3x + 2 = 0 \ (mod\ 10)$. (6 marks)

Question five

a) Find x such that $x \equiv 3 \ mod\ 5$ and $x \equiv 7 \ mod\ 11$ (5 marks)

b) Find one pair of positive integers a, b such that

i) $ab(a + b)$ is not divisible by 7

ii) $(a + b)^7 - a^7 - b^7$ is divisible by 7^7 . Justify your answer. (15 marks)