(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 306**

COURSE TITLE: **GROUP THEORY II**

DATE: 21/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages.** Please Turn Over.

QUESTION ONE (20MARKS)

a). Define the following

i). Normal series (2marks)

ii). Composition series (2marks)

iii). Soluble Group (3marks)

b). Define the following

i). Division Algorithm (2marks)

ii). Group (4marks)

iii). Subgroup (3marks)

iv). Normal subgroup (1mark)

c). Let G be a group and $H \geq G$. If H and G/H are soluble, show that G is soluble. (7marks)

d). If G is a soluble group and $H \leq G$, show that H is also soluble (6marks)

QUESTION TWO (20MARKS)

a) Define the following

i). Direct product (3marks)

ii). The dimension of G over Zp. (3marks)

b) If G is a finite a belian P-group, show that $p^n G/P^{n+1}G$ is an elementary abelian P-group (5marks)

c) If $(m.n) = 1$, show that $Zmn \cong Zm \times Zn$ (9marks)

QUESTION THREE (20MARKS)

a). Define the following

i). Sylow-p subgroup (2marks)

ii). Conjugate (2marks)

iii). Normalizer (2marks)

iv) The dihedral group (2marks)

v). The quaternion group (2marks)

b). Let p be a sylow –p subgroup of G. Show that $N_G(P)/P$ has no element whose order is a power of p except for the identity (6marks) c). Let P be sylow-P subgroup of G and let $g \in G$ have order o power of P. If $g^{-1}pg = p$, show that $p \in P$ (4MARKS)

QUESTION FOUR (20MARKS)

a). Let G be a cyclic group of order 12 generated by a. Determine

i). G (3marks)

ii). $\langle a^4 \rangle$ (2marks)

iii). $\langle a^3 \rangle$ (2marks)

b). State Euler phi function (3marks)

c). Let G be a cyclic group of order n generated by a. Show that G has $Q(n)$ generators (5marks)

d) Let G be a cyclic group of order n. If d divides n, show that the number of elements of order d in G is $Q(d)$. It is 0 otherwise (5marks)

QUESTION FIVE (30MARKS)

a). Using the symmetries of rectangles construct the Klein 4 group (4marks)

b). Show that the Klein 4 group is abelian (6marks)

c). Let $G\langle a \rangle$ have order n. Show that for each k dividing n. G has a unique subgroup of order K, namely $\langle a^{nk} \rangle$ (4marks)

d). Show that every subgroup of acyclic group is cyclic (8marks)