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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: **MAT 306**

COURSE TITLE: GROUP THEORY

DATE: 31/07/18 **TIME: 2 PM -4 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE: COMPULSORY (30 MARKS)

a) Define, using relevant examples, the following terms
   i) Conjugacy class (2 marks)
   ii) Class equation (2 marks)
   iii) Abelian group (2 marks)
   iv) P-subgroup (2 marks)

b) Show how the class equation of the Quaternion group is determined. (8 marks)

c) State the four Sylow theorems. (4 marks)

d) Determine the number of abelian groups of order 72 and show them clearly in a table. (10 marks)

QUESTION TWO (20 MARKS)

a) Identify all the p-Sylow subgroups of $Z/(12)$ (10 marks)

b) Show that every group of order p^2 is abelian where p is prime. (10 marks)

QUESTION THREE (20 MARKS)

a) Identify the elements of 2-Sylow subgroup of $SL_2(Z/(3))$ (10 marks)

b) State and prove the Jordan-Holder theorem. (10 marks)

QUESTION FOUR (20 MARKS)

a) Identify the conjugate classes of the Dihedral group D_8 , the set of the symmetries of a square, hence write its class equation. (10 marks)

b) Show a finite group G has a p-Sylow subgroup for every prime p and any p-subgroup of G lies in a p-Sylow subgroup of G . (10 marks)

QUESTION FIVE (20 MARKS)

a) Show that for each prime p , the p-Sylow subgroups of G are conjugate. (10 marks)

b) Let $G = S_3$, permutation group on three elements, identify its class equation from the Conjugacy class. (10 marks)