(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 305**

COURSE TITLE: **GROUP THEORY I**

DATE: 12/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a) Define the following

i. Normal subgroup (2marks)

ii. Quotient group (2marks)

iii. Conjugate (2marks)

iv. The center of the group (2marks)

v. The centralizer (2marks)

b) If G is a group and $a \in G$, show that $a * a = a$ implies $a = e$ (3marks)

c) Define the following

i. Identity (2marks)

ii. Inverse (2marks)

d) If S is a subset of the group G, show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab^{-1} \in S$ (5marks)

e) Define the following

i. A subgroup (2marks)

ii. Proper subgroup (2marks)

iii. Trivial subgroup (1mark)

iv. Cyclic subgroup (2marks)

QUESTION TWO (20MARKS)

a) Define the following

i. Right coset (2marks)

ii. The order of the group (2marks)

iii. Lagranges theorem (2marks)

b) Let S be a subgroup of the group G and let $a, b \in G$. Show that $Sa = Sb$ if and only if $ab^{-1} \in S$ (6marks)

c) Show that cosets are either identical or disjoint (6marks)

d) If $|G| = P$ is prime. Show that G is cyclic (2marks)

QUESTION THREE (20MARKS)

a) Show that every subgroup of a cyclic group is cyclic (6marks)

b) Define the Euler Phi Function (2marks)

c) Let G be a cyclic group of order n generated by a. show that G has $\emptyset(n)$ generators (4marks)

d) Compute the following with the Euler phi function

i. $\emptyset(40)$ (3marks)

ii. $\emptyset(300)$ (3marks)

iii. $\emptyset(6^3)$ (3marks)

QUESTION FOUR (20MARKS)

a) Define the following

i. Permutation(3marks)

ii. Group action (3marks)

iii. Orbit(2marks)

iv. Stabilizer (3marks)

b) Show that every permutation can be written as the product of transposition(3marks)

c) Let G act on Ω. If $x \in \Omega$, show that $|x^G| = \frac{|G|}{|G_x|}$ (6marks)

QUESTION FIVE (20MARKS)

a) Show that the subgroup N of G is a normal subgroup if and only if $g^{-1}Ng \subseteq N$ for all $g \in G$ (5marks)

b) Show that if N is a normal subgroup of G. Then the coset of N form a group. If G is finite this group has order $|G: N|$ (5marks)

c) Define the following

i. Binary operation (1mark)

ii. A group (3marks)

d) Write down the multiplication table for the set of matrices

$$G = \left\{a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, b = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, c = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, d = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\right\}$$ (3marks)

e) Write down the multiplication table for the set of complex numbers

$G = \{1, i, -1, -1\}$ under multiplication (3marks)