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(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 305

COURSE TITLE: GROUP THEORY I

DATE: 02/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1 (30 marks)

a). Define the following

i) Subgroup (3marks)

ii) Isomorphic groups (2marks)

iii) the center of a group (3marks)

iv). Order of a group (2marks)

b). Let S be a subgroup of the group G and let $a, b \in G$. Show that $Sa = Sb$ if and only if $ab^{-1} \in S$ (5marks)

c). Show that every subgroup of a cyclic group is cyclic. (10marks)

d). Show that the set of complex numbers $G = \{I, i, -I, -i\}$ under multiplication is a group (5marks)

Question 2 (20marks)

a). Define the following

i). Group (3marks)

ii). Subgroup generated by X (2marks)

iii). The Kernel and the Image (3marks)

iv). Symmetric group (2marks)

b). If S is a subset of the group G, show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab^{-1} \in S$ (5marks)

c). Show that cosets are either identical or disjoint (5marks).

Question 3 (20marks)

a). Show that every permutation can be written as the product of transposition (3marks)

b). If S is a subset of the finite group G, show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab \in S$ (10marks)

c). Let $G = \langle a \rangle$ have order . Show that for each K dividing n, G has a unique subgroup of order K namely $\langle a^{n/k} \rangle$. (7marks)

Question 4 (20 marks)

a). Define the following

i). Orbit (2marks)

ii). Stabilizer (2marks)

iii). Graph (2marks)

iv). Centralizer in a group (2marks)

b). Let $s_4 = sym(1,2,3,4)$ show that s_4 acts on the set of ordered parts (6marks)

c). Let x be an element of the finite group G. Show that the number of conjugates of x is the index of $C_{G(X)}$ in G. That is, $|K(x)| = \left[G: C_{G(X)}\right]$ (7marks)

Question 5 (20marks)

a). Define the following

i). Transposition (1mark)

ii). Even permutation (2marks)

iii). Normal subgroup (2marks)

iv). Quotient group (2marks)

b). State the Division Algorithm Lemma (3marks) c). Let$\pi = B_1B_2,,,,,,,,,B_n = y_1y_2,,,,,y_m$ be two factorizations of the permutation π where the B_i and Y_j are transpositions. Show that either n and m are both even or they are both odd. (4marks)

d). Show that the subgroup N of G is a normal subgroup of G if and only if $g^{-1}Ng \subseteq N$ for all $g \in G$ (6marks)