KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS)

COURSE CODE: MAT 305

COURSE TITLE: GROUP THEORY I

DATE: 10/01/18   **TIME**: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1 (30 marks)

a). Define the following

    i) A group   (3marks)

    ii) Isomorphic groups   (2marks)

    iii) A graph   (3marks)

    iv). Symmetric group   (2marks)

b). If S is a subset of the group G, show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab^{-1} \in S$ (5marks)

c). If S is a subset of the finite group G, show that S is a subgroup of G if and only if S is nonempty and whenever $a, b \in S$, then $ab \in S$ (10marks)

d). Show that the set of complex numbers $G = \{I, \dot{I}, - I, -\dot{I}\}$ under multiplication is a group

                                                            (5marks)

Question 2 (20marks)

a). Define the following

    i). Subgroup   (3marks)

    ii). Subgroup generated by X   (2marks)

    iii). The Kernel and the Image   (3marks)

    iv). Order of a group   (2marks)

b). Let S be a subgroup of the group G and let $a, b \in G$. Show that $Sa = Sb$ if and only if $ab^{-1} \in S$ (5marks)

c). Show that cosets are either identical or disjoint   (5marks).

Question 3 (20marks)

a). State the Division Algorithm Lemma   (3marks)

b). Show that every subgroup of a cyclic group is cyclic.   (10marks)

c). Let $G = \langle a \rangle$ have order n. Show that for each K dividing n, G has a unique subgroup

of order K namely $\langle a^{n/k} \rangle$.   (7marks)

Question 4 (20 marks)

a). Define the following

      i). Normal subgroup   (2marks)

      ii). Quotient group   (2marks)

      iii). the center of a group   (2marks)

      iv). Centralizer in a group   (2marks)

b). Show that the subgroup N of G is a normal subgroup of G if and only if $g^{-1}Ng \subseteq N$ for all
    $g \in G$ (6marks)

c). Let x be an element of the finite group G. Show that the number of conjugates of x is the
    index of $C_{G(x)}$ in G. That is, $|K(x)| = \left[G : C_{G(x)}\right]$ (7marks)

Question 5 (20marks)

a). Define the following

      i). Transposition   (1mark)

      ii). Even permutation   (2marks)

      iii). Orbit   (2marks)

      iv). Stabilizer   (2marks)

b). Show that every permutation can be written as the product of transposition   (3marks)

c). Let $\pi = B_1 B_2 ,,,,,,,,, B_n = y_1 y_2 ,,,,, y_m$ be two factorizations of the permutation π

where the B_i and Y_j are transpositions. Show that either n and m are both

even or they are both odd. (4marks)

d). Let $s_4 = sym(1,2,3,4)$ show that s_4 acts on the set of ordered parts (6marks)