(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 304**

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 22/09/17 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

UNIT CODE: MAT 304 **UNIT TITLE**: COMPLEX ANALYSIS I MAIN EXAM

INSTRUCTIONS: Answer question one and any other two

QUESTION ONE (Compulsory)

a) Evaluate $(-1+i)^{1/3}$ and represent first three solutions graphically (8 marks)

b) State the De Moivre's theorem and use it to solve $Z^5 = 3 - 4i$ (5 marks)

c) Show that an analytic function is also harmonic. (5 marks)

d) Describe the singularities of the function $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ (5 marks)

e) Evaluate $\int \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$ (7 marks)

QUESTION TWO

a) State the five theorems of continuity. (5 marks)

b) Prove that

i) $e^{z_1}e^{z_2} = e^{z_1+z_2}$ (4 marks)

ii) $|e^z| = e^x$ (3 marks)

iii) $e^{z+2\pi k} = e^z, k = 0, \pm1, \pm2, \ldots$ (3 marks)

c) Prove that $\lim_{z\to i} \frac{3z^4-2z^3+8z^2-2z+5}{z-i} = 4 + 4i$ (5 marks)

QUESTION THREE

a) Evaluate the following using the theorems of limits

i) $\lim_{z\to 1+i} z^2 - 5z + 10$ (3 marks)

ii) $\lim_{z\to -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$ (3 marks)

iii) $\lim_{z\to 2e^{\pi i/3}} \frac{z^3+8}{z^4+4z^2+16}$ (4 marks)

b) Is the function $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ continuous at $z = i$? (5 marks)

c) Define an analytic function and state the necessary condition for a function $w = f(z)$ to be analytic in a region $\mathbb{R}$ (5 marks)

QUESTION FOUR

a) State any five rules of differentiation. (5 marks)

b) Define the singularity of a function and using a relevant example in each case, describe any four types of singularity. (10 marks)

c) Using the definition of derivative, find the derivative of $w = f(z) = z^2 - 2z + 1$ at $z = z_o$ and $z = -1$ (5 marks)

QUESTION FIVE

a) For each of the following functions , locate and name the singularities in the finite z plane and determine whether they are isolated singularities or not

i) $f(z) = \frac{z}{(z^2+4)^2}$ (3 marks)

ii) $f(z) = \sec\left(\frac{1}{z}\right)$ (3 marks)

iii) $f(z) = \frac{\ln(z-2)}{(z^2+2z+2)^4}$ (4 marks)

b) Evaluate $\int \bar{z}dz$ from $z = 0$ to $z = 4 + 2i$ along the curve given by $z = t^2 + it$

(5 marks)

c) Let $f(z)$ be analytic inside and on a simple closed curve C except at a pole a of order m inside C. Prove that the residue of $f(z)$ at a is given by

$$a_{-1} = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}$$ (5 marks)