(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND

BACHELOR OF SCIENCE

COURSE CODE: MAT 304

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 01/08/18   **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE

(a) Define the following terms:
   (i)   Limit of a function   (1 mark)
   (ii)   Continuity   (1 mark)
   (iii)   Simply connected region   (1 mark)
   (iv)   Multi-connected region   (1 mark)

(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.   (7 marks)

(c) Show that the function $e^x(\cos y + i \sin y)$ is an analytic function, find its derivative expressing it in terms of z.   (6 marks)

(d) Find the image of the circle $|z - 1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$.   (4 marks)

(e) Find the first four terms of the Taylor series expansion of the complex variable function $f(z) = \frac{z+1}{(z-3)(z-4)}$ about $z = 2$ .find the region of convergence.   (5 marks)

QUESTION TWO

(a) If $w = \phi + i\psi$ represents the complex potential for an electrical field and
$$\psi = x^2 - y^2 + \frac{x}{x^2 + y^2},$$
determine the function ϕ .   (8 marks)

(b) Given $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $= x + iy$, find $f(z)$ in terms of z .   (12 marks)

QUESTION THREE

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c = $ constant.   (7 marks)

(b) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Using *Milne Thompson* method, find the corresponding analytic function $f(z)$ in terms of z.   (13 marks)

QUESTION FOUR

(a) State and prove Cauchy's integral theorem.   (10 marks)

(b) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$ using Cauchy's Integral Formula.   (10 marks)

QUESTION FIVE

(a) Using residue theorem, evaluate $I = \oint_C \frac{z^2+4}{z^3+2z^2+2z} \, dz$, where C is $|z + 1 - i| = 1$.

(9 marks)

(b) Consider the function $f(z) = 4x + y + i(-x + 4y)$, show that the function $f(z)$ is differentiable

(11 marks)