(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

(MATHEMATICS) AND BACHELOR EDUCATION

COURSE CODE: **MAT 304**

COURSE TITLE: COMPLEX ANALYSIS I

DATE: 28/4/16 **TIME:** 8 AM -10 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

KIBABII UNIVERSITY

THIRD YEAR SECOND SEMESTER EXAMINATION 2015/2016

(MAIN CAMPUS)

MAT 304: COMPLEX ANALYSIS I

Instruction: Attempt question ONE and any other TWO questions

Question 1: Compulsory (30 marks)

a). (i) State Cauchy's integral formula (2mks)

(ii)Using (i) or otherwise evaluate

$$\oint_C \frac{e^{-z}}{z+1}dz \quad C : |z| = |2|$$

(3mks)

b). Prove that

i). $e^{ix} + e^{-ix} = 2\cos x$ (2mks)

ii). $e^{ix} - e^{-ix} = 2i\sin x$ (2mks)

c). Find the $\lim_{z\to 10+i}\left\{\frac{z^3+2z-4+i}{z+i+2}\right\}$ (2mks)

d). Simplify the following

$$\frac{\left[\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right]^7}{\left[\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi\right]^{\frac{3}{2}}}$$

(4mks)

e). Classify the singularities of the following complex functions

(i) $f(z) = \dfrac{1}{z^3(z^2 - iz + 6)}$ (3mks)

(ii) $f(z) = e^{\frac{1}{z-1}}$ (2mks)

f). State and prove Laurent series (10mks)

Question 2 (20mks)

a) State and prove Taylor's theorem for an analytic function (10mks)

b) Using (a) or otherwise write the Taylor series expansion of $f(z) = \frac{1}{z^2 - 9}$ about $z = 1$ (10mks)

Question 3 (20mks)

a) Let $f(z)$ be analytic in a simply connected region **R**. Suppose further that **C** is any simple closed curve in **R**, prove that $\oint_C f(z)dz = 0$ (7mks)

b) State the Residue theorem (3mks)

c) Evaluate $\oint_C (z - \text{Re}(z))dz \quad C : |z| = 2$ (10mks)

Question 4 (20mks)

a) (i) What is an analytic function? (2mks)
(ii) Differentiate between harmonic functions and conjugate harmonic functions (3mks)

b) Given the real function $u(x, y) = e^x \cos y$, show that u satisfies a Laplace equation and hence find its conjugate harmonic function and write the function $f(z)$ (10mks)

c) Evaluate $\oint_C \frac{1}{z^n} dz \quad C$: is a unit circle centre origin oriented counterclockwise (positive direction) (5mks)

Question 5 (20mks)

a) Find the analytic function $w = f(z)$ from its known real part $u(x, y) = 2e^x \cos y$ and the condition $f(0) = 2$ (6mks)

b) Show that $u(x, y) = e^{2x}(x \cos 2y - y \sin 2y)$ is harmonic (5mks)

c) Evaluate $\oint_C (x^2 + ixy)dz$ from $A(1, 1)$ to $B(2, 4)$ along the curve $x = t$ and $y = t^2$ (5mks)

d) Find the image of the triangle ABC given $A(0, 0), B(3, 0)$ and $C(2, 3)$ under the transformation $f(z) = z(2 + 2i) - (4 + 2i)$ (4mks)