(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 303**

COURSE TITLE: LINEAR ALGEBRA III

DATE: 11/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30marks)

(a) Let A be a square matrix over a complex field $\mathbb{C}$. Suppose λ is an eigenvalue of A^2, show that $\sqrt{\lambda}$ or $-\sqrt{\lambda}$ is an eigen value of A. (4mks)

(b) Given $= \begin{bmatrix} \frac{1+i}{2} & \frac{1+i}{2} \\ \frac{1-i}{2} & \frac{-1+i}{2} \end{bmatrix}$. Show that A is unitary (3mks)

(c) Given $A = \begin{bmatrix} 2 & 1+i \\ 1-i & 3 \end{bmatrix}$

i. Show that A is a normal matrix (1mk)

ii. Diagonize A (6mks)

(d) i. Define a bilinear form (2mks)

ii. Let V be a vector space of dimension n over K and let $\{\phi_1 \ldots \ldots \ldots \ldots . \phi_n\}$ be any basis on a dual space V^*. Show that $\{f_{ij}: i, j = 1 \ldots .. n\}$ is a basis of $B(V)$ where f_{ij} is defined by $f_{ij}(u, v) = \phi_i(u)\phi_j(v)$ and $\dim B(V) = n^2$. (5mks)

(e) Let A be the a matrix such that $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 7 & -5 \\ 2 & -5 & 8 \end{bmatrix}$

i. Find a non-singular matrix P such that $D = P^T AP$.

ii. Find the signature of A. (5mks)

(f) . Find the adjoint of $G: \mathbb{C}^3 \to \mathbb{C}^3$ defined by

$G(x, y, z) = [2x + (1-i)y, (3+2i)x - 4iz, 2ix + (4-3i)y - 3z]$ (4mks)

QUESTION TWO (20 MKS)

(a) Suppose $A \neq I$ is a square matrix for which $A^3 = I$. Determine wether or not A is similar to a diagonal matrix if A is a matrix over

i. the real field

ii. the complex field (4mks)

(b) Find the adjoint of the operator $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ such that $F(x, y, z) = (3x + 4y - 5z, 2x - 6y + 7z, 5x - 9y + z)$. (4mks)

(c) Define a nilpotent operator (2mks)

(d) Let A and B be matrices such that:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

i. Verify that the matrices are nilpotent of index 3. (3mks)

ii. Find nilpotent matrices M_A and M_B respectively in canonical form that are similar to A and B respectively (4mks)

(e) Let A be an $n \times n$ matrix over K. Show that the mapping f defined by $f(X, Y) = X^T AY$ is a bilinear form on K^n. (3mks)

QUESTION THREE (20 MKS)

(a) Define a Hermitian form. (2mks)

(b) Given $q(x, y)$ is a quadratic form such that $q(x, y) = 3x^2 + 2xy - y^2$ with linear substitutions $x = s - 3t$, $y = 2s + t$.

i. Write $q(x, y)$ in matrix notation and find the matrix A representing $q(x, y)$. (2mks)

ii. Re-write the linear substitution using matrix notation and find the matrix P corresponding to the substitution. (3mks)

iii. Find $q(s, t)$ using direct substitution. (3mks)

iv. Find $q(s,t)$ using matrix notation. (2mks)

(c) Show that $q(x,y) = ax^2 + bxy + cy^2$ is positive definite iff $a > 0$ and the discriminant $D = b^2 - 4ac > 0$. (4mks)

(f) Determine wether each of the following is positive definite

i. $q(x,y,z) = x^2 + 2y^2 - 4xz - 4yz + 7z^2$ (2mks)

ii. $q(x,y,z) = x^2 + y^2 + 2xz + 4y^2 + 3z^2$ (2mks)

QUESTION FOUR (20MKS)

(a) Let T, T_1, T_2 be linear operators on V and let $\in K$. Show that

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$ (2mks)

(ii) $(kT)^* = \overline{k}T^*$ (2mks)

(iii) $(T_1T_2)^* = T_2^*T_1^*$ (2mks)

(iv) $(T^*)^* = T$ (2mks)

(b) Let ϕ be a linear functional on a finite dimensional inner product space.Show that there exists a unique vector $u \in V$ such that $\phi(v) = \langle v, u \rangle$ for every $v \in V$. (6mks)

(c) Let ϕ be an eigenvalue of a linear operator $T(V)$. Show the following.

i. If $T^* = T^{-1}$ then $|\lambda| = 1$ (2mks

ii. If $T^* = T$ then λ is real. (2mks)

iii. If $T = S^*S$ with S non-singular, then λ is real and positive. (2mks)

QUESTION FIVE (20 MKS)

(a) Let f be the bilinear form on $\mathbb{R}^3$ defined by $f[(x_1,x_2),(y_1,y_2)] = 3x_1y_1 - 2x_1y_2 + 4x_2y_1 - x_2y_2$. Find

i. the matrix A of f in the basis $\{u_1 = (1,1), u_2 = (1,2)\}$ (3mks)

ii. the matrix B of f in the basis $\{v_1 = (1,-1), v_2 = (3,1)\}$ (3mks)

iii. the change of basis matrix P from $\{u_i\}$ to $\{v_i\}$ and verify that $B = P^T AP$. (4mks)

(b) i. Distinguish between bilinear forms which are alternating and skew symetric . (2mks)

ii. Show that an alternating bilinear form is also symetric. (3mks)

(e) Given $= \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{bmatrix}$. Find a non-singular matrix P such that $D = P^T AP$. (5mks)