(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 303**

COURSE TITLE: **LINEAR ALGEBRA III**

DATE: 01/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

(a).Define the following terms

(i). Define an orthogonal matrix (1 mk)

(ii). Complex inner product (5 mk)

(iii). Hermitian matrix (1 mk)

(b). (i).Show that$(1-21i, 6-9i, 13)$ and $(1+3i, -2-i, 5)$ are eigenvectors of a hermitian matrix. (3 mks)

(ii). Find all complex scalars k for which u and v are orthogonal in $\mathbb{C}^3$.

$u = (2i, i, 3i) v = (i, 6i, k)$. (2 mks)

(c). (i). If A and B are complex matrices and $\bar{A}$ and $\bar{B}$ the complex conjugates, show that$\overline{AB} = \overline{A}\overline{B}$.

(2 mks)

(ii). Illustrate the above relation for matrices P and Q where

$P = \begin{bmatrix} -5i & 4 \\ 2-i & 1+5i \end{bmatrix} Q = \begin{bmatrix} 4i & 2-3i \\ 2+3i & 1 \end{bmatrix}$ (4 mks)

(d). Prove that if A is an $n \times n$ orthogonal matrix, then the row and well as column vectors of A forms an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product (5 mks)

(e). Find the quadratic form associated with matrix $\begin{bmatrix} 5 & -3 \\ -3 & 8 \end{bmatrix}$ and show that it is positive definite. (4 mks)

(f). Show that orthogonally diagonalizable matrix must be symmetric. (3 mks)

QUESTION TWO (20 MKS)

(a). Show that if H is a hermitian matrix, then its eigenvectors from different eigenspaces are orthogonal. (4mks)

(b).i. Show that is $A = \begin{bmatrix} \frac{1}{2} & -\frac{i}{2} & \frac{i-1}{2} \\ \frac{i}{2} & \frac{1}{2} & \frac{1+i}{2} \\ \frac{i+1}{2} & \frac{i-1}{2} & 0 \end{bmatrix}$ a unitary matrix (3 mks)

ii. If U is a unitary matrix, show that it is isometric. (4 mks) (c). if λ is an eigenvalue of real $n \times n$ matrix A, and if x is the corresponding eigenvector, then $\bar{\lambda}$is also an eigenvalue of A and $\bar{x}$ is a corresponding eigenvector. (3mks)

(d). Given that $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{bmatrix}$, find a matrix H such that $H = P^T DP$. (6mks)

QUESTION THREE (20 MKS)

(a).(i). Show that an inner product defined by $f(u,v) = \sum_{i=1}^{n} u_i \bar{v_i}$ is a bilinear form.(4mks)

(ii). Show that $q(x,y) = ax^2 + bxy + cy^2$ is positive definite iff $a > 0$ and the discriminant $D = b^2 - 4ac > 0$. (4mks)

(b).(i) If $u, v, w \in \mathbb{C}^n$ $\alpha \in \mathbb{C}$, show that $\langle w, u - \alpha v \rangle = \langle w, u \rangle - \bar{\alpha}\langle w, v \rangle$ (3 mks)

(ii). Given that $u, v \in \mathbb{C}^n$ define the Eclidean inner product and the norm. (2 mks)

(iii) Determine the Euclidean norm of the vectors $u = (1 + i, 2i, 7 - 2i)$ and $v = (-2i, 4,4 + i)$. (3 mks)

(c). Let A be a linear operator over a complex vector space V with k a complex no. Prove that

i. $(A^*)^* = A$ (2 mks)

ii. $(kA)^* = \bar{k}A^*$ (2 mks)

QUESTION FOUR (20 MKS)

(a). If $v, u \in \mathbb{C}^n$, and$\bar{u}$ and $\bar{v}$ are complex conjugates of u and v, for $\alpha \in \mathbb{C}$,show that

(i). $\bar{\bar{u}} = u$ (3mks)

(ii). $\overline{\alpha u} = \bar{\alpha}\bar{u}$ (3 mks)

(i). $\overline{u - v} = \bar{u} - \bar{v}$ (3 mks)

(b).Show that $A = \begin{bmatrix} i & -i & 1 \\ -1 & 1 & i \\ 0 & 0 & 1 + i \end{bmatrix}$ is not diagonolizable. (5 mks)

(c) Let A be an $n \times n$ matrix over K. Show that the mapping f defined by $f(X,Y) = X^T AY$ is a bilinear form on K^n. (4 mks) (d). Find inner product $\langle u, v\rangle$ of the vectors given by $u = (2+3i, -4i, 6i-1)$ and $v = (2-3i, 4i, -6i-1)$.(2mks)

QUESTION FIVE (20 MKS)

(i). let$u, v \in \mathbb{C}^n$,and$k \in \mathbb{C}$, if $f(u, v) = \langle u, v\rangle$ is an inner product, show that $\langle u, kv\rangle = \bar{k}\langle u, v\rangle$.

(2 mks)

(ii).Define an orthogonal matrix, hence show that $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is orthogonal. (3 mks)

(iii). Orthogonally diagonalize matrix A if $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$. (15 mks)