(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 303**

COURSE TITLE: LINEAR ALGEBRA III

DATE: 19/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30marks)

(a). Define a nilpotent matrix, hence show that $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a nilpotent matrix hence state the index of nilpotency. (3 mks)

(b). (i). Let $\mathbb{C}^n$ be a complex vector space $u \in \mathbb{C}^n$ and $k \in K$, K A scalar field. If $\bar{u}$ denotes the conjugates of u and v respectively, show that $\overline{ku} = \bar{k}\bar{u}$. (3 mks)

(ii). Let $u, v \in \mathbb{C}^n$ with $u = (3 - 4i, 2 + i, -6i)$ and $v = (1 + i, 2 - i, 4)$. Find the Euclidian norm $\|u\|$ of u and the Euclidean inner product $\langle u, v \rangle$. (4 mks)

(c). (i). Given that x is the eigenvector of a nonzero matrix A corresponding to an eigenvector λ. Explain why $x \neq 0$. (2 mks)

(ii). Determine the eigenvalues and eigenvectors for the following matrix

$\begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}$ (4 mks)

(d). If A is an orthogonal matrix,

(i). $\det(A) = 1$ or $\det(A) = -1$. (2 mks)

(ii). $\|Ax\| = \|x\|$ for $x \in \mathbb{R}^n$ (3 mks)

(e). i.Make a change of variable to transform the quadratic form $Q(x) = x_1^2 - 8x_1x_2 - 5x_2^2$ into a quadratic form with no cross-product terms. (5mks)

ii. Show that an inner product defined by $f(u, v) = \sum_{i=1}^{n} u_i \overline{v_i}$ is a complex valued bilinear form. (4 mks)

QUESTION TWO (20 MKS)

(a) Determine all possible Jordan Canonical forms J for a linear operator $T: V \rightarrow V$ whose characteristic polynomial $\Delta(t) = (t - 2)^5$ and whose minimum polynomial $m(t) = (t - 2)^2$.

(4mks)

(b). Let A and B be linear operators on complex vector space V such that $A: V \rightarrow V$ and $: V \rightarrow V$. If k a complex no. Prove that

i. $(A^*)^* = A$ (2 mks)

ii. $(A + B)^* = A^* + B^*$ (2 mks)

iii. $(kA)^* = \bar{k}A^*$ (2 mks)

iv. $(AB)^* = B^*A^*$ (2 mks)

(c). Prove that Eigenvalues of real symmetric matrices are real and the eigenvectors are orthogonal.

(5 mks)

(b). Show that $U = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2}(1+i) & \frac{1}{2}(1+i) \end{bmatrix}$ is a Unitary matrix. (3 mks)

QUESTION THREE (20 MKS)

(a).(i).What is a bilinear form? (2 mks)

(ii). Classify the following quadratic forms and positive definite, negative definite or indeginite.

$g(x, y) = 4x^2 - 2x_1x_2 + 4x_2^2$ (2 mks)

$f(x, y) = 5x^2 + 4x_1x_2 + 5x_2^2$ (2 mks)

(b). (i). Define a hermitian matrix. (1 mk)

(ii). Show that of H is a hermitian matrix, then its eigenvectors from different eigenspaces are orthogonal. (4 mks)

(iii). Find the eigenvalues of matrix A.

$$A = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}$$ (4 mks)

(b). Let $\langle Ax \cdot Ay \rangle = \langle x \cdot y \rangle$ for all $x, y \in \mathbb{R}^n$. Show that A is orthogonal. (5 mk)

QUESTION FOUR (20 MKS)

(a). If U is unitary then, show that

(i). $U^*U = 1$ (1 mk)

(ii). U is an isometry. (4 mk)

(iii). Under what conditions is the matrix $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & c \\ 0 & b & 0 \end{bmatrix}$ unitary? (3 mks)

(b).(i) Show that matrix $C = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$ is orthogonal. (3 mks)

(c). Prove that if A is an $n \times n$ orthogonal matrix, then the row and well as column vectors of A forms an orthonormal set in $\mathbb{R}^n$ with the Euclidean inner product (5 mks)

(d). if λ is an eigenvalue of real $n \times n$ matrix A, and if x is the corresponding eigenvector, then $\bar{\lambda}$is also an eigenvalue of A and $\bar{x}$ is a corresponding eigenvector. (4 mks)

QUESTION FIVE (20 MKS)

(i). Find a quadratic form corresponding to the following symmetric matrix

$B = \begin{bmatrix} 4 & -5 & 7 \\ -5 & -6 & 8 \\ 7 & 8 & -9 \end{bmatrix}$ (2 mks)

(ii). Show that if A is orthogonally diagonalizable, it must be symmetric. (3 mks)

(iii). Orthogonallydiagonalize matrix A, $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix}$ (15mks)