(15)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 302**

COURSE TITLE: REAL ANALYSIS III

DATE: 19/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One:(30 marks)

(a) Define the following terms:
(i)Refinement of a partition. [2 marks]
(ii)Function of a bounded variation. [2 marks]
(i)Step function. [2 marks]

(b) The partition P_n of [0,1]is given by $\{0 < \frac{1}{n} < \frac{2}{n} < ..... < \frac{n-1}{n} < 1\}$. If $f(x) = x^3$, find:
(i) $\underline{S}_{P_n}(f)$ [2 marks]
(ii) $\overline{S}_{P_n}(f)$ [2 marks]
(iii) $\int_0^1 f(x)dx$ [1 marks]

(c) Show that the Dirichlet function defined by f(x) = 1 for all $x \in [0, 1]$ is rational and f(x)= 0 if $x \in [0, 1]$ is irrational is not Riemann integrable. **[4 marks]**

(d) If $f : [a, b] \longrightarrow \Re$ is continuous on [a,b]. Show that $f \in R[a, b]$. **[5 marks]**

(e) Let $f : [a, b] \longrightarrow \Re$ be a function of bounded variation. Show that $v(f, [a, b]) = 0$ if and only if f is a constant on [a,b]. **[5 marks]**

(f) Show that if $a_n \geq 0$ and $a_{n+1} \geq a_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges. **[5 marks]**

Question Two:(20 marks)

(a) Discuss the pointwise convergence of the sequence of functions (x^n). **[6 marks]**

(b) Show that if α is non- decreasing and f,g are Riemann - Stieltjes integrable with respect to α, then:

$$\int_a^b (f + g)d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$$

[6 marks]

(c) Let $f : [a, b] \longrightarrow \Re$ be a function, $\{x_i : 0 \leq i \leq n\}$ be a partition of [a,b] and $\{y_i : 0 \leq i \leq m\}$ be a partition of [a,b] such that $\{x_i : 0 \leq i \leq n\} \subseteq \{y_i : 0 \leq i \leq m\}$. Show that:

$$\sum_{i=1}^{n} \mid f(x_i) - f(x_{i-1}) \mid \leq \sum_{i=1}^{m} \mid f(y_i) - f(y_{i-1}) \mid$$

[8 marks]

Question Three:(20 marks)

(a) Prove that if p^* is a refinement of partition p, then $l(p, f, \alpha) \leq l(p^*, f, \alpha)$ and $u(p, f, \alpha) \geq u(p^*, f, \alpha)$. **[8 marks]**

(b) Show that a sequence f_n of bounded functions on $A \subseteq \Re$ converges uniformly on A to f if and only if $\|f_n - f\| \longrightarrow 0$. **[5 marks]**

(c) If $\{E_n : n \in \mathbf{N}\}$ is a pairwise disjoint collection of members of lebesgue measurable set, then show that for any $X \subseteq \mathbf{R}$:

$$m^*(X \cap (\cup_{i=1}^{\infty} E_i)) = \sum_{i=1}^{\infty} m^*(X \cap E_i)$$

[7 marks]

Question Four:(20 marks)

(a) Show that a function defined by

$$f(x) = \begin{matrix} 0 when x is rational \\ 1 when x is irrational \end{matrix} .$$

is not Riemann integrable **[6 marks]**

(b) Prove that if f is increasing on [a,b] then $v(f, [a, b] = f(b) - f(a)$. **[6 marks]**

(c) Consider the partition P_n of $[0,1]$ given by $\{0 < \frac{1}{n} < \frac{2}{n} < .... < \frac{n-1}{n} < 1\}$.
If $f(x) = x^2$, find:
(i) $\underline{S}_{P_n}(f)$ **[2 marks]**
(ii) $\overline{S}_{P_n}(f)$ **[2 marks]**
(iii) $\int_0^1 f(x^2)dx$ **[2 mark]**

Question Five: (20 marks)

(a) Prove that any function is Riemann integrable if and only if for a given $\varepsilon > 0$, there exists a partition P such that

$$| u(P,f) - l(P,f) | < \epsilon$$

[7 marks]

(b) Show that every continuous function is integrable. **[7 marks]**

(c) Let $\{f_n\}$ be a sequence of continuous functions. If $\{f_n\}$ converges uniformly to f on [a,b] then show that f is continuous and $lim_{n \longrightarrow \infty} \int_a^b f_n(x)dx = \int_a^b f(x)$ **[6 marks]**

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