(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 302

COURSE TITLE: REAL ANALYSIS III

DATE: 10/08/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One:(30 marks)

(a) Define the following terms:

(i)Countably subadditive. [2 marks]

(ii)Function of a bounded variation. [2 marks]

(i)Partition of an interval. [2 marks]

(b) The partition P_n of [0,1]is given by $\{0 < \frac{1}{n} < \frac{2}{n} < ..... < \frac{n-1}{n} < 1\}$. If $f(x) = x^3$, find:

(i) $\underline{S}_{P_n}(f)$ [2 marks]

(ii) $\overline{S}_{P_n}(f)$ [2 marks]

(iii) $\int_0^1 f(x)dx$ [1 marks]

(c) Show that the function f defined by

$$f(x) = \begin{cases} 0, ifxisirrational \\ 1, ifxisrational \end{cases}$$

is not of bounded variation on any interval. [6 marks]

(d) Let s and t be two simple functions defined on a measurable set E of finite Lebesgue measure. Show that

$$\int_E (s+t)d\mu = \int_E s d\mu + \int_E t d\mu$$

[5 marks]

(e) If f is Riemann-Stieltjes integrable with respect to α on [a,b], then it is Riemann integrable on each sub-interval $[c, d] \subseteq [a, b]$. If $c \in [a, b]$, show that:

$$\int_a^b f d\alpha = \int_a^c f d\alpha + \int_c^b f d\alpha$$

[3 marks]

(f) Show that if $a_n \geq 0$ and $a_{n+1} \geq a_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{k=0}^{\infty} 2^k a_{2^k}$ converges. [5 marks]

Question Two:(20 marks)

(a) Suppose $f \in R(\alpha)$ on [a,b] ,$m \leq f \leq M$, and is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on [a,b]. Show that $h \in R(\alpha)$ on [a,b]. **[6 marks]**

(b) Show that if α is non- decreasing and f,g are Riemann - Stieltjes integrable with respect to α, then:

$$\int_a^b (f+g)d\alpha = \int_a^b f d\alpha + \int_a^b g d\alpha$$

. **[6 marks]**

(c) State Comparison Test. **[3 marks]**

(c) Let A and B be sets. If $m^*(B) = 0$, then show that $m^*(A \bigcup B = m^*(A)$ for all subsets A of $\mathbf{R}$. **[5 marks]**

Question Three:(20 marks)

(a) Proof that if p^* is a refinement of partition p, then $l(p, f, \alpha) \leq l(p^*, f, \alpha)$ and $u(p, f, \alpha) \geq u(p^*, f, \alpha)$. **[9 marks]**

(b) Suppose that if f is a function of bounded variation on [a,b] then proof that it is so on any subinterval of [a,b]. **[4 marks]**

(c) If $\{E_n : n \in \mathbf{N}\}$ is a pairwise disjoint collection of members of lebesgue measurable set, then show that for any $X \subseteq \mathbf{R}$:

$$m^*(X \cap (\cup_{i=1}^{\infty} E_i)) = \sum_{i=1}^{\infty} m^*(X \cap E_i)$$

. **[7 marks]**

Question Four:(20 marks)

(a) (i)State the necessary condition for convergence and the necessary and sufficient condition for convergence. **[2 marks]**

(ii) Show that the function $f(x) = x^2$ is uniformly continuous on [-1,1]. **[6 marks]**

(b) Proof that if f is increasing on [a,b] then $v(f, [a, b] = f(b) - f(a)$. **[6 marks]**

(c) Consider the partition P_n of $[0, 1]$ given by $\{0 < \frac{1}{n} < \frac{2}{n} < .... < \frac{n-1}{n} < 1\}$.
If f(x)= x, find:
(i) $\underline{S}_{P_n}(f)$ **[2 marks]**
(ii) $\overline{S}_{P_n}(f)$ **[2 marks]**
(iii) $\int_0^1 f(x)dx$ **[2 mark]**

Question Five: (20 marks)

(a) Let $f : [a, b] \longrightarrow \mathbf{R}$ be a function and $\{x_i : 0 \leq i \leq n\}$ be a partition of [a,b]; $\{y_i : 0 \leq i \leq m\}$. be a partition of [a,b] whereby $\{x_i : 0 \leq i \leq n\} \subseteq \{y_i : 0 \leq i \leq n\}$. Show that

$$\sum_{i=1}^{n} | f(x_i) - f(x_{i-1}) | \leq \sum_{i=1}^{m} |f(y_i) - f_{(i-1)}|$$

[6 marks]

(b) Suppose that f is bounded on [a,b], has only finitely many points of discontinuities in I =[a,b] and that the monotonically increasing function α is continuous at each point f. Show that $f \in R(\alpha)$. **[9 marks]**

(c) Let $\{f_n\}$ be a sequence of continuous functions. If $\{f_n\}$ converges uniformly to f on [a,b] then show that f is continuous and $lim_{n \longrightarrow \infty} \int \int_a^b f_n(x)dx = \int_a^b f(x)$ **[5 marks]**

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