(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 301

COURSE TITLE: REAL ANALYSIS II

DATE: 17/01/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One:(30 marks)

(a) Carefully define the terms with respect to a metric space: (i) Limit point (ii) Compact set (iii) Point of contact **[6 marks]**

(b) Show that if (X, ρ) is a metric space then (x_n) is a sequence of points which is Cauchy then (x_n) is bounded. **[5 marks]**

(c) Let $(X_1, \rho_1)(X_2, \rho_2)$ be metric spaces.If $X = X_1 \times X_2$ and ρ is defined on $X_1 \times X_2$ by

$$\rho\{(X_1, X_2), (Y_1, Y_2) = \rho_1(X_1, Y_1) + \rho_2(X_1, Y_2)\}$$

Show that (X, ρ) is a metric space. **[8 marks]**

(d) Show that for any subset E of a metric space (X, ρ), E^o is an open set. **[5 marks]**

(e) Show that a set X in a metric space is compact if and only if every infinite subset of X has at least one limit point in X. **[6 marks]**

Question Two:(20 marks)

(a) Show that if M and N are subsets of X and (X, ρ) is a metric space, then $M \subseteq N$ implies that $M^d \subseteq N^d$. **[4 marks]**

(b) Prove that if (X, λ) and (Y, ρ) are metric spaces, then a function $f : X \to Y$ is continuous at the point $x \in X$ if and only if for any open neighbourhood V of f(x) in Y, there is a neighbourhood of U of x such that $f(U) \subseteq V$. **[6 marks]**

(c) Show that every converging sequence in a metric space (X, ρ) is a Cauchy sequence. **[5 marks]**

(d) Show that every open neighbourhood in $(X, \rho$ is always open with respect to the metric space. **[5 marks]**

Question Three:(20 marks)

(a) Let (X, ρ) be a metric space and $E \neq \emptyset \subseteq X$. Prove that a point y is a point of contact of E if and only if there is a sequence (x_n) of points in E such that x_n converges in (X, ρ) to y. **[7 marks]**

(b) Show that $\overline{E} = E \bigcup E^d$. **[6 marks]**

(c) (i) Define the term closed subset in a metric space. **[2 marks]**
(ii) Show that a set E is open in (X, ρ) if and only if E^c is closed in (X, ρ) . **[5 marks]**

Question Four:(20 marks)

(a) Let A be a fixed non empty subset of a metric space (X, d). Show that the real-valued function f defined on X by $f(x) = D(x, A)$ is continuous. **[5 marks]**

(b) Let (X, ρ) be a metric space and E be a subset of X. Show that a point $a \in X$ is a limit point of E with respect to (X, ρ) if and only if every neighbourhood N(a), contains infinitely distict points of E. **[7 marks]**

(c) State and prove Cauchy- Schwarz inequality . **[8 marks]**

Question Five: (20 marks)

(a) Show that if (X, ρ) is a metric space, then no sequence in X can converge to more than one point. **[5 marks]**

(b) (i) Define the term complete metric space. **[1 mark]**
(ii) Let X be a non-void set and define the function ρ on $X \times X$ by $\rho(x, y) = 1$ if and only if $x \neq y$ and $\rho(x, y) = 0$ whenever x = y. Show that (X, ρ) is a metric space. **[8 marks]**

(c) (i) Define the term interior point of a set in a metric space. **[2 marks]** (ii)Let (X, ρ) be a metric space. Show that $\emptyset$ and a set X are always open in (X, ρ). **[4 marks]**

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