(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

COURSE CODE: MAT 301

COURSE TITLE: REAL ANALYSIS II

DATE: 05/12/18 **TIME:** 8.00-10.00AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question One:(30 marks)

(a) Define the following terms:
(*i*) Interior point of a set [2 marks]
(*ii*) Point of contact of a set [2 marks]
(*iii*) Complete metric space [2 marks]

(b) Let $(X_1, \rho_1), (X_2, \rho_2)$ be metric space and if $X = X_1 \times X_2$, define ρ on $X \times X$ by $\rho((X_1, X_2)(Y_1, Y_2)) = \rho_1(X_1, Y_1) + \rho_2(X_2, Y_2)$. Show that X, ρ is a metric space. **[7 marks]**

(c) Show that a set E is open if and only if $E = E^0$, where E^0 is an interior point of E. **[6 marks]**

(d) Show that the limit of a convergent sequence is unique in a metric space **[7 marks]**

(e) Proof that the closure of the set X is equal to the union of the set X and the derived set of X **[4 marks]**

Question Two:(20 marks)

(a) Show that every open neighbourhood is an open set. **[7 marks]**

(b) Show that derived set is closed in a metric space. **[7 marks]**

(c) Proof that a set E is closed in a metric space if and only if its complement is open in a metric space. **[6 marks]**

Question Three:(20 marks)

(a) Show that the interior of a set E is open **[6 marks]**

(b) State and proof Cauchy - Schwarz inequality **[9 marks]**

(c) Let X, ρ be a metric space and (x_n) be a sequence of points which is cauchy. Then (x_n) is bounded i.e the range (x_n) is a bounded subset**[5** marks]

Question Four:(20 marks)

(a) Show that if (X, ρ) is a metric space, then:
(i) $\emptyset$,X are open in (X, ρ)
(ii) If $(E_\alpha : \alpha \in \Omega)$ is a family of open subsets of (X, ρ) then, $\bigcup_{\alpha\in\Omega} E_\alpha$ is open in (X, ρ)
(iii) If $\{E_1, E_2, ..., E_n\}$ is a family of open subsets of (X, ρ), then: $\bigcap_{i=1}^{n}$ is open in $(X, \rho$ **[12 marks]**

(b) Show that a subset A of a real number is closed if and only if whenever $a_n \in A$ for all$n \in \mathbf{N}$, and $a_n \longrightarrow c$, we have $c \in A$. **[8 marks]**

Question Five: (20 marks)

(a) Define the following terms with respect to a metric space:
(i) Open set **[2 marks]**
(ii) Limit point **[2 marks]**

(b) Show that a subset A of X is closed if and only if it contains all its limit points. **[4 marks]**

(c) Let (X, ρ_1) and (Y, ρ_2) be metric spaces, then a function $f : X \longrightarrow Y$ is continuous at the point $x \in X$ if and only if for any open neighbourhood V of f(x) in Y, there is a neighbourhood U of x such that $f(U) \subseteq V$ **[12 marks]**

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