(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 226

COURSE TITLE: CLASSICAL MECHANICS

DATE: 01/08/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

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QUESTION ONE (COMPULSORY) (30MARKS)

a) i) Define mechanics (1mk)

ii) Mechanics is further subdivided into a number of sections. State and distinguish them. (6mks)

b) If $\mathbf{A} = (2xy + z^3)\mathbf{i} + (x^2 + 2y)\mathbf{j} + (3xz^2 - 2)\mathbf{k}$, find:

i) $\nabla \times \mathbf{A}$, (3mks)

ii) A scalar function ϕ such that $\mathbf{A}=\nabla\phi$. (4mks)

c) A particle of constant mass m moves in a space under the influence of a force of a field $\boldsymbol{F}$. Assuming that at times t_1 and t_2 the velocity is $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ respectively; prove that the work done is the change in kinetic energy. (5mks)

d) Show that a force field $\boldsymbol{F}$ defined by$\boldsymbol{F} = (y^2z^3 - 6xz^2)\boldsymbol{i} + 2xyz^3\boldsymbol{j} + (3xy^2z^2 - 6x^2z)\boldsymbol{k}$ is a conservative force field. (5mks)

e) i) State the principle of conservation of angular momentum (2mks)

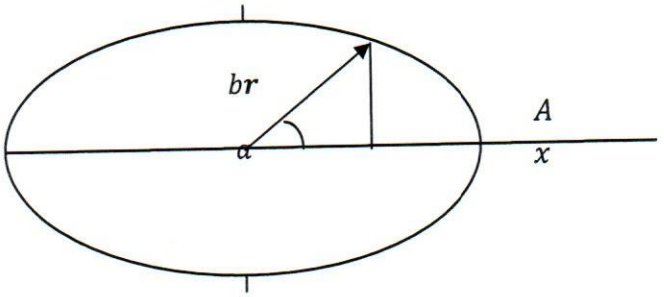
ii) Is the force field given by $\boldsymbol{F} = x^2yz\boldsymbol{i} - xyz^2\boldsymbol{k}$ conservative? (4mks)

QUESTION TWO (20MARKS)

A particle of mass m moves in the xy plane so that its position vector is$\boldsymbol{r} = a\cos\omega t\boldsymbol{i} + b\sin\omega t\boldsymbol{j}$, where a, $b$and ω are positive constants and $a > b$.

By

B



(a) Show that the particle moves in an ellipse. (3marks)

(b) Show that the force acting on the particle is always directed toward the origin. (3marks)

(c) Show that the force field is conservative. (3marks)

(d) Find the kinetic energy of the particle at points A and B. (4marks)

(e) Find the work done by the force field in moving the particle from A to B. (3marks)

(f) Find the total energy of the particle and show that it is constant. (4marks)

QUESTION THREE (20 MARKS)

a) A particle of mass 2 moves in a force field depending on time t given by $\mathbf{F} = 24t^2\mathbf{i} + (36t - 16)\mathbf{j} - 12t\mathbf{k}$. Assuming that at $\boldsymbol{t = 0}$ the particle is located at $\mathbf{r_0} = \mathbf{3i - j + 4k}$ and has a velocity $\mathbf{v_0} = \mathbf{6i + 15j - 8k}$, find:

i) the velocity (4marks)

ii) The position at any time$t$. (4marks)

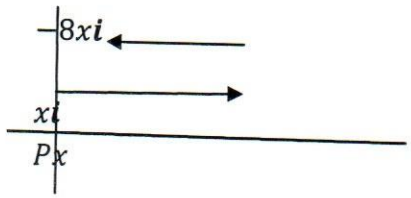
b) Show that $\mathbf{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field.(4marks)

i) Find the potential. (4marks)

ii) find the work done in moving an object in this field from $(1, -2, 1)$ to$(3, 1, 4)$. (4marks)

QUESTION FOUR (20 MARKS)

a) A particle $\boldsymbol{P}$ of mass 2 moves along the x axis attracted toward origin$O$ by a force whose magnitude is numerically equal to $8x$ as shown below.



If it is initially at rest at $x = 20$, find:

i) The differential equation and initial conditions describing the motion. (3marks)

ii) The position of the particle at any time, (4marks)

iii) The speed and velocity of the particle at anytime (2marks)

iv) The amplitude, period and frequency of the vibration. (3marks)

b) (i) Show that the function $Acos\omega t + Bsin\omega t$ can be written as $Ccos(\omega t - \phi)$ where $C = \sqrt{A^2+B^2}$ and $\phi = \tan^{-1} B/A$. (5marks)

(ii) find the amplitude, period and frequency of the function in b (i) (3marks)

QUESTION FIVE (20 MARKS)

A projectile is launched with initial speed v_0 at an angle α with the horizontal. Find:

(a) The position vector at any time. (5marks)

(b) The time it takes to reach the maximum height. (3marks)

(c) The maximum height reached. (3marks)

(d) The time of flight back to the earth and the range. (4marks)

(e) Show that the path of the projectile is a parabola. (3marks)

Prove that the range of the projectile is a maximum when the launching angle $\alpha = 45^{\circ}$ (2 marks)