(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 206**

COURSE TITLE: ALGEBRAIC STRUCTURES II

DATE: 21/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Define the following

   i. A real sequence (2 marks)

   ii. Limit of a sequence (3 marks)

   iii. Null sequence (2 marks)

   iv. Bounded sequence (2 marks)

b. Show that every convergent sequence is bounded (5 marks)

c. Show that the limit of a convergent sequence is unique (7 marks)

d. Let $\mathbb{R} \subseteq X \times Y$be a binary relation from X to Y. Let $A, B \subseteq X$be subsets. Show that ;

   i. If $A \subseteq B$then $R(A) \subseteq R(B)$ (4 Marks)

   ii. $R(A \cup B) = R(A) \cup R(B)$ (6 Marks)

QUESTION TWO (20 MARKS)

a. Define the following

   i. Complementary relation (2 marks)

   ii. Inverse relation (2 marks)

   iii. Mathematical induction (2 marks)

b. Prove that this rule of exponents is true for every natural number n:$(ab)^n = a^n b^n$

(7 marks)

c. Determine whether 225 is divisible by 2,3,4,5,6.9 and 10 (7marks).

QUESTION THREE (20 MARKS)

a. Define the following

   i. Prime number (2 marks)

   ii. Fundamental theorem of arithmetic (2marks)

   iii. Greatest common divisor (2 marks)

b. Show that a composite integer n has a prime factor less than or equal to $\sqrt{n}$ (5 marks)

c. Show that there is an infinite number of prime numbers (6 marks)

d. Show that for positive integers a and b we have $ab = gcd(a.b).lcm(a.b)$ (3 marks)

QUESTION FOUR (20 MARKS)

a. Define the following;

   i. Group (3 marks)

   ii. Abelian group (2 marks)

   iii. The subgroup criterion (2 marks)

b. Show that $(Z, +)$ is a group (3 marks)

c. Show that $[e, (1,2,3), (1,3,2)] \leq s_3$ (5 marks)

d. Let G be a group and $a\epsilon G$. Show that <a> is a subgroup of G. (4 Marks)

QUESTION FIVE (20 MARKS)

a. Define the following

   i. Ring (5marks)

   ii. Field (2marks)

b. State and prove the two properties of fields (8 marks)

c. Show that Z_3 is a field (3 marks)

d. State three properties of rings (3 marks)