(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 206**

COURSE TITLE: ALGEBRAIC STRUCTURES II

DATE: 11/10/18 **TIME:** **11.30 AM -1.30 PM**

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a. Let $\mathbb{R} \subseteq X \times Y$be a binary relation from X to Y. Let $A, B \subseteq X$be subsets. Show that ;

   i. If $A \subseteq B$then $R(A) \subseteq R(B)$ (4 Marks)

   ii. $R(A \cup B) = R(A) \cup R(B)$ (6 Marks)

b. Prove that this rule of exponents is true for every natural number n:$(ab)^n = a^n b^n$

   (7 marks)

c. Determine whether 225 is divisible by 2,3,4,5,6.9 and 10 (7marks).

a. Define the following

   i. Prime number (2 marks)

   ii. Fundamental theorem of arithmetic (2marks)

   iii. Greatest common divisor (2 marks)

QUESTION TWO (20 MARKS)

a. Show that a composite integer n has a prime factor less than or equal to $\sqrt{n}$ (5 marks)

b. Show that there is an infinite number of prime numbers (6 marks)

c. Show that for positive integers a and b we have $ab = gcd(a.b).lcm(a.b)$ (3 marks)

d. Define the following

   i. Complementary relation (2 marks)

   ii. Inverse relation (2 marks)

   iii. Mathematical induction (2 marks)

QUESTION THREE (20 MARKS)

a. Show that every convergent sequence is bounded (5 marks)

b. Show that the limit of a convergent sequence is unique (6 marks)

c. Define the following;

   i. Group (3 marks)

ii. The subgroup criterion (2 marks)

iii. Abelian group (2 marks)

d. Show that $(Z, +)$ is a group (3 marks)

QUESTION FOUR (20 MARKS)

a. Define the following

i. A real sequence (2 marks)

ii. Limit of a sequence (3 marks)

iii. Null sequence (2 marks)

iv. Bounded sequence (2 marks)

b. State and prove the two properties of fields (8 marks)

c. Show that Z_3 is a field (3 marks)

QUESTION FIVE (20 MARKS)

a. Define the following

i. Ring (5marks)

ii. Field (2marks)

b. State three properties of rings (3 marks)

c. Show that $[e, (1,2,3), (1,3,2)] \leq s_3$ (5 marks)

d. Let G be a group and $a \epsilon G$. Show that <a> is a subgroup of G. (5 Marks)