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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 206**

COURSE TITLE: **ALGEBRAIC STRUCTURES II**

DATE: 31/07/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

a. Define the following

i. Complementary relation (2 marks)

ii. Prime number (2 marks)

i. A real sequence (2 marks)

ii. Abelian group (2 marks)

iii. Inverse relation (2 marks)

b. Let $\mathbb{R} \subseteq X \times Y$ be a binary relation from X to Y. Let $A, B \subseteq X$ be subsets. Show that ;

If $A \subseteq B$ then $R(A) \subseteq R(B)$ (4 Marks)

c. Show that a composite integer n has a prime factor less than or equal to $\sqrt{n}$ (6 marks)

d. Show that every convergent sequence is bounded (5 marks)

e. Show that $(Z, +)$ is a group (5 marks)

QUESTION TWO (20 MARKS)

a. Define the following

iii. Mathematical induction (2 marks)

iv. Fundamental theorem of arithmetic (2marks)

i. Greatest common divisor (2 marks)

ii. Limit of a sequence (2 marks)

b. Let $\mathbb{R} \subseteq X \times Y$ be a binary relation from X to Y. Let $A, B \subseteq X$ be subsets. Show that ;

$R(A \cup B) = R(A) \cup R(B)$ (6 Marks)

c. In $\mathbb{Z}_3$ show that every nonzero element is its own inverse (3marks)

d. State three properties of rings (3marks)

QUESTION THREE (20 MARKS)

a. Define the following

iii. Null sequence (2 marks)

iv. Bounded sequence (2 marks)

b. Prove that this rule of sum of consecutive cubes is true for every natural number

$n: 1^3+2^3+3^3+..........+n^3 = \frac{n^2(n+1)^2}{4}$ (8marks)

c. Show that there is an infinite number of prime numbers (5 marks)

d. Show that for positive integers a and b we have $ab = gcd(a.b).lcm(a.b)$ (3 marks)

QUESTION FOUR (20 MARKS)

a. Define the following;

i. Group (3 marks)

ii. The subgroup criterion (2 marks)

b. Show that the limit of a convergent sequence is unique (6 marks)

c. Let G be a group and $a\epsilon G$. Show that <a> is a subgroup of G. (4 Marks)

d. Let $\mathbb{C}^* = \mathbb{C}\backslash\{0\}$,the set of all nonzero complex numbers.

Show that $(\mathbb{C}^*,\times)$is a group (5marks)

QUESTION FIVE (20 MARKS)

a. Define the following

i. Divisibility test method (2marks)

ii. Field (2marks)

iii. Ring (5marks)

b. Construct the cayley table for multiplication in $\mathbb{Z}_6$ (3 marks)

c. From b above determine which elements have inverses (2 marks)

d. Determine whether 7168 is divisible by 2,3,4,5,6 and 8 (6 marks)