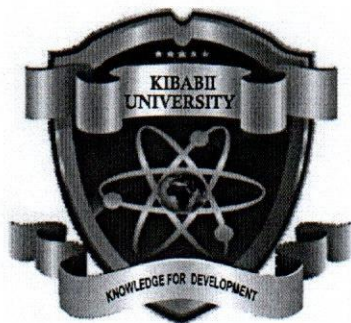


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(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE
MATHEMATICS

COURSE CODE: MAT 206

COURSE TITLE: ALGEBRAIC STRUCTURES II

DATE: 31/07/18

TIME: 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE (30 MARKS)

- a. Define the following (2 marks)
- i. Complementary relation (2 marks)
 - ii. Prime number (2 marks)
 - i. A real sequence (2 marks)
 - ii. Abelian group (2 marks)
 - iii. Inverse relation (2 marks)
- b. Let $\mathbb{R} \subseteq X \times Y$ be a binary relation from X to Y . Let $A, B \subseteq X$ be subsets. Show that ; (4 Marks)
If $A \subseteq B$ then $R(A) \subseteq R(B)$
- c. Show that a composite integer n has a prime factor less than or equal to \sqrt{n} (6 marks)
- d. Show that every convergent sequence is bounded (5 marks)
- e. Show that $(\mathbb{Z}, +)$ is a group (5 marks)

QUESTION TWO (20 MARKS)

- a. Define the following (2 marks)
- iii. Mathematical induction (2marks)
 - iv. Fundamental theorem of arithmetic (2 marks)
 - i. Greatest common divisor (2 marks)
 - ii. Limit of a sequence (2 marks)
- b. Let $\mathbb{R} \subseteq X \times Y$ be a binary relation from X to Y . Let $A, B \subseteq X$ be subsets. Show that ; (6 Marks)
 $R(A \cup B) = R(A) \cup R(B)$ (3marks)
- c. In \mathbb{Z}_3 show that every nonzero element is its own inverse (3marks)
- d. State three properties of rings

QUESTION THREE (20 MARKS)

- a. Define the following
- iii. Null sequence (2 marks)
 - iv. Bounded sequence (2 marks)
- b. Prove that this rule of sum of consecutive cubes is true for every natural number
- $$n: 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \quad (8 \text{ marks})$$
- c. Show that there is an infinite number of prime numbers (5 marks)
- d. Show that for positive integers a and b we have $ab = \gcd(a, b) \cdot \text{lcm}(a, b)$ (3 marks)

QUESTION FOUR (20 MARKS)

- a. Define the following;
- i. Group (3 marks)
 - ii. The subgroup criterion (2 marks)
- b. Show that the limit of a convergent sequence is unique (6 marks)
- c. Let G be a group and $a \in G$. Show that $\langle a \rangle$ is a subgroup of G . (4 Marks)
- d. Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$, the set of all nonzero complex numbers.
Show that (\mathbb{C}^*, \times) is a group (5 marks)

QUESTION FIVE (20 MARKS)

- a. Define the following
- i. Divisibility test method (2 marks)
 - ii. Field (2 marks)
 - iii. Ring (5 marks)
- b. Construct the cayley table for multiplication in \mathbb{Z}_6 (3 marks)
- c. From b above determine which elements have inverses (2 marks)
- d. Determine whether 7168 is divisible by 2,3,4,5,6 and 8 (6 marks)