(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 204**

COURSE TITLE: REAL ANALYSIS I

DATE: 20/09/17 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of **3 Printed Pages**. Please Turn Over.

QUESTION 1 : COMPULSORY (30 MARKS)

a) Given $a \in \mathbb{R}$ and that $a \neq 0$ prove that a^{-1} exists . **(5mks)**

b) Given a functions f , g such that $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$.
Define $f(x) = x^3 - x^2$ and $g(x) = 4x + 5$. Find

(i) $fg(x)$ **(3 mks)**

(ii) $gf(x)$ **(3 mks)**

(iii) Hence the value of $fg(2)$ and $gf(2)$ **(4mks)**

c) If x and y are positive real numbers the prove that $x < y$ iff $x^2 < y^2$ **(7mks)**

d) Given that $a, b, v, u \in \mathbb{R}$, show that if $v + a = a$ the $v = 0$ and if $ub = b$ for $b \neq 0$ then $u = 1$ **(6 mks)**

e) Define the term Supremum of a set **(2mks)**

QUESTION 2 : (20 MARKS)

a)If a and b are real numbers , prove the triangle inequality $|a| + |b| \geq |a + b|$ **(8mks)**

b) Given that A and B are subsets of a universal set U then prove that
$(A \cap B)^c = A^c \cup B^c$ **(8 mks)**

c)Find the supremum and infimum of the set

$A = \left\{\frac{2+n}{n} \quad : n = 1, 2, 3, \dots \dots \dots\right\}$ **(4mks)**

QUESTION 3 : (20 MARKS)

a) Let a and b be any two distinct rational numbers in $\mathbb{R}$and that$a < b$. Show that we can find we can find another rational number q between a and b . That is $a < q < b$ **(8mks)**

b) $\forall\, a, b, c \in \mathbb{R}$ Prove that $a^2 + b^2 + c^2 \geq ab + ac + bc$ **(8mks)**

c) If $xz = yz$ and that $z \neq 0$, then show that $x = y$ **(4 mks)**

QUESTION 4 : (20 MARKS)

a) For any $x \in \mathbb{R}$ and that $x \neq 0$. Show that $x^2 > 0$ **(4mks)**

b) Show that $\sqrt{18}$is irrational **(8mks)**

c) Given that the set $X = \{1, 2, 3\}$ Find $\mathbb{P}(X)$ the power set of X **(8mks)**

QUESTION 5 : (20 MARKS)

a) Define the term rational number **(2 mks)**

b) Prove that:

i) The union of an arbitrary collection of open subsets in$\mathbb{R}$ is open. **(4mks)**

ii) The union of an arbitrary collection of closed subsets in$\mathbb{R}$ is closed. **(4 mks)**

c) If $\forall\, n \in \mathbb{N}$, the set E is countable then prove that $\bigcup_{n=1}^{\infty} E_n$ is also countable. **(10mks)**