(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 204**

COURSE TITLE: REAL ANALYSIS I

DATE: 10/10/18 **TIME:** 3 PM -5 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE : (30 MARKS)

a) Define the following terms
   (i). Rational number set (2 mks)
   (ii). Open sets (2 mks)
   (iii). Sets difference (1mk)
   (iv). Order axiom (2 mks)

b) Let $\mathbb{F}$ be a field and $x, y \in \mathbb{F}$. Show that $|x + y| \leq |x| + |y|$.(5mks)

c) Let $\delta = 0.0002$. Given that $N(x, \delta)$ is a neighbourhood of x for $x \in \mathbb{R}$

   (i). Find a neighborhood contained in $N(x, \delta)$. (2 mks)

   (ii). Find an closed set contained in the neighborhood of (i) above. (3 mks)

d) Show that an infinite subset of a countable set is countable (4mks)

e) Give the precise definition of a continuous function, hence, show that $f(x) = 5 - 6x$ is continuous at $x = 3$. (6 mks)

f) Let A be a closed unbounded subset of the set $\mathbb{R}$. Provide an example of A explaining the two features it possess, closedness and unboundedness. (3 mks)

QUESTION TWO (20 MARKS)

a) Define an onto function given an example (2mks)

b) Show that arbitrary union collection of open sets $\{B_i \subset \mathbb{R}, i \in \Lambda\}$ is open. (5 mks)

c) (i). Prove that there is no rational number x such that $x^2 = 3$. (7 mks)
   (ii). State the completeness axiom. (2 mks)

d) Let f be a function from a space X to another Y.
   (i). Define inverse of f. (1 mks)
   (ii). If $f(x) = \frac{5-x}{2+3x}$ find $f^{-1}(x)$. (3 mks)

QUESTION THREE (20 MARKS)

a) Differentiate between the supremum and infimum of a subset A of a field $\mathbb{F}$. (4 mks)

b) State the first principal of induction hence use it too show that for $n \geq 1$, $1^2 + 2^2 + 3^2 + \cdots + k^n = \frac{n}{6}(n + 1)(2n + 1) n \in \mathbb{N}$. (8mks)

c) Let $A$and B be two finite sets. Show that $(A \cup B)^c = A^c \cap B^c$. (8 mks)

QUESTION FOUR (20 MARKS)

a) Let A and B be two sets such that $A = [-2,3]$ and $B = (0,2]$. Find $A \cap B$. (2 mks)

b) Let $\{E_1, E_2, \dots E_n\}$ be a collection of closed sets. Show that the finite intersection, $\bigcap_{i=1}^{n} E_n$ is closed. (5 mks)

c) (i). What is an ordered field? (3 mks)

(ii). Given $\mathbb{F}$ is an ordered field and $a, b \in \mathbb{F}$ show that $|a| \leq b$ implies $-b \leq a \leq b$. (3 mks)

(iii). Let $\mathbb{F}$ be an ordered field. Define a metric d on the field as

$$d(x,y) = f(x) = \begin{cases} 1 & x \neq y \\ 0 & x = y \end{cases} \text{ for } x, y \in \mathbb{F}$$

Show d is a metric. (7mks)

QUESTION FIVE (20 MARKS)

a) Define the term Cartesian product of sets A and B, hence find the Cartesian product of the two sets if $A = \{-4, -2, 5\}$ and $B = \{3,1,2\}$. (5mks)

b) Let $P = (0,1]$ and $Q = \left(\frac{1}{n}, n^2\right), n \in \mathbb{N}$. Find the minimum, maximum, infimum and supremum of the two sets. (8 mks)

c) What is a countable set (1 mks)

d) Let $X = \{2,3,5,7,11,18\}$.

(i). Categorize the set as finite or infinite. Explain. (2 mks)

(ii). Categorize X as bounded and unbounded given reasons. (4 mks)