(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: MAT 204

COURSE TITLE: REAL ANALYSIS I

DATE: 30/07/18 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

QUESTION ONE : (30 MARKS)

a) Define the following terms

   (i). Bounded set (2 mks)

   (ii). Closed set (2 mks)

   (iii).Disjoint sets (1 mk)

   (iv).Ordered field (2 mks)

b) Let $\mathbb{F}$ be a field and $x, y \in \mathbb{F}$. Show that$|x| - |y| \leq |x - y|$. (5mks)

c) Let $A = \{E_i \subset \mathbb{R}, i \in I\}$ be a collection of open sets. Show that arbitrary union of members of A is open. (4 mks)

d) Prove that for any pair $a , b \in \mathbb{R}$ we can find infinitely many real numbers (5 mks)

e) Identify the point(s) at which the function $f(x) = \frac{1}{\cos x}$ has discontinuities. (4 mks)

f) Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two functions on subspaces of a field $\mathbb{F}$.
   If$f(x) = \frac{1}{2-x}$ and $g(x) = \frac{4x+1}{5}$, find$(f \circ g)^{-1}(x)$. (5 mks)

QUESTION TWO :(20 MARKS)

a) Let $A = \{x: -1 < x < 0\}$ be a subset of the real number field $\mathbb{R}$.

   (i). Categorize the set as either closed or open, explain. (2 mks)

   (ii). Determine the closure, $\bar{A}$, of A. Explain your answer. (2 mks)

   (iii). Is A bounded? Explain (3 mks)

b) State the completeness axiom. (2 mks)

c) Let $A$and B be two finite sets. Show that $(A \cap B)^c = A^c \cup B^c$. (6 mks)

d) Prove that there is no rational number x such that $x^2 = 2$. (5 mks)

QUESTION THREE :(20 MARKS)

a) (i).Define the terms supremum and infimum of a subset A of a field $\mathbb{F}$. (4 mks)

(ii). Determine the infimum, supremumof the following sets.

(4 mks)

$A = \left(-1, \frac{1}{n}\right) n \in \mathbb{N}$

(4 mks)

$B = \left[1, \frac{2+n}{n}\right] n \in \mathbb{N}$

b) Using principles of mathematical induction , show that for $n \geq 1$, $8^n - 3^n$ is divisible by 5 for $n \in \mathbb{N}$. (8 mks)

QUESTION FOUR : (20 MARKS)

a) Define a composition of two functions? (2 mks)

b) Show that the limit of the function $f(x) = \frac{3}{x^2-4}$ as x approaches 2 does not exists. (5 mks)

c) Let $\mathbb{F}$ be an ordered field. Define a metric d on the field as$d(x,y) = |x - y|$for $x, y \in \mathbb{F}$. Show d is a metric. (6 mks)

d) Show that an infinite subset of a countable set is countable (6 mks)

e) What is a relation between two sets $A$and B? (1 mks)

QUESTION FIVE :(20 MARKS)

a) (i). What is a countable set? (1 mk)

(ii). Differentiate between finite and infinite set. (2mks)

(ii). Show that the set of integers is countably infinite. (4 mks)

b) Let $\{A_1, A_2, \ldots A_n\}$ be a collection of closed sets. Show that the finite intersection,$\bigcap_{i=1}^{n} A_n$is closed. (5mks)

c) Let $\mathbb{F}$ be an ordered field. Prove that a nonempty subset A of$\mathbb{F}$ has at most one least upper bound and at most one greatest lower bound. (4 mks)

d) Differentiate between injective and subjective functions giving examples in each case. (4 mks)