(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 204**

COURSE TITLE: REAL ANALYSIS I

DATE: 12/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30MARKS)

a. Define the following

   i. Open neighbourhood (3marks)

   ii. Open set (2marks)

   iii. Countable Set (2marks)

b. For any two finite sets A and B, show that $(A \cup B)^C = A^C \sqcap B^C$ (11 marks)

c. If $\{E_1, E_2, \dots, E_n\}$ is any finite collection of closed subsets of X w.r.t.(X,d), show that $\bigcap_{i=1}^{n} E_i$ is also closed (5marks)

d. Show that $|a + b| \leq |a| + |b|$ (7marks)

QUESTION TWO (20MARKS)

a. Define the following

   i. Bounded set (2marks)

   ii. The completeness theorem (2marks)

   iii. Supremum (2marks)

b. Show that if x and y are positive real numbers, then x< y iff $x^2 < y^2$ (9marks)

c. Let S be a non-empty set of real numbers with sup say b. show that $\forall\ a < b\ \exists\ x \in S$ such that $a < x \leq b$ (5marks)

QUESTION THREE (20MARKS)

a. Define the following

   i. Surjective (2marks)

   ii. Injective (2marks)

b. Consider the function $f(1,-\infty) \to (0,1)$ defined by $f(x) = \frac{x-1}{x+1}$. Show that f posses an inverse $f^{-1}(y) = \frac{y+1}{1-y}$ (4marks)

c. Show that the empty set $\emptyset$ is always open (5marks)

d. Suppose that A and β_λ for all $\lambda \in \Lambda$ are given sets. Show that

$$A \cup (\cap \beta_\lambda) = \cap (A \cup \beta_\lambda)$$ (7marks)

QUESTION FOUR (20MARKS)

a. Define the following

   i. Removable discontinuity (2marks)

   ii. Infinite discontinuity (2marks)

   iii. Finite discontinuity (2marks)

b. Show that a set S of real numbers is bounded iff $\exists k \in \mathbb{R}$ such that $|x| \leq k$ for all $x \in S$ (8marks)

c. Show that $f(x) = \begin{cases} \frac{2x-6}{x-3} & when\ x \neq 3 \\ 2 & when\ x = 3 \end{cases}$ is continuous at $x = 3$ (6marks)

QUESTION FIVE (20MARKS)

a. Define the following

  i. Composition of a function (2marks)

  ii. Inverse of a function (2marks)

  iii. Cardinality (2marks)

b. Show that $\emptyset, X$ are always closed in (X, d) (3marks)

c. Show that if $x \in \mathbb{R}$ then $|xy| = |x||y|$ (4marks)

d. Show that an infinite subset of a countable set is countable (3marks)

e. Let (X, d) be a metric space. Show that the union of any arbitrary family of subsets open in (X, d) is open in (X, d). (4marks)