(Knowledge for Development)

# KIBABII UNIVERSITY

MAIN EXAMINATION

## UNIVERSITY EXAMINATIONS

## 2016/2017 ACADEMIC YEAR

## SECOND YEAR SECOND SEMESTER

## FOR THE DEGREE OF BACHELOR OF SCIENCE

### MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

**COURSE CODE:** MAT 204

**COURSE TITLE:** REAL ANALYSIS I

**DATE:** 03/12/18 **TIME:** 11.30-1.~~30PM~~

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**INSTRUCTIONS TO CANDIDATES**

Answer Question One and Any other TWO Questions

TIME: 2 Hours

**QUESTION ONE (COMPULSORY) (30 MARKS)**

a) Show that if $x \neq 0$, then $x^{-1} \neq 0$ and $x^{-1}$ is unique. (3mks)

b) For every $x \neq 0$, show that $x^2 > 0$, hence show that $1 > 0$. (3mks)

c) Let (S, <) be an ordered set and E a subset of S, if the least upper bound of E (lubE) and the greatest lower bound of E (glbE) exist. Show that i) the lubE is unique (4 mks)

ii) the glb.E is unique. (4 mks)

d) Show that $\sqrt{3}$ is an irrational number. (4mks)

e) State the completeness axiom for $\Re$ (2mks)

f) Let A be a nonvoid subset of $\Re$ which is bounded above. Define a set B by $B = \{-x; x \in A\}$, show that B is bounded below and -sup A= inf B. (4mks)

g) If a and b are given real numbers such that for every real number $\varepsilon > 0$, $a \leq b + \varepsilon$, show that $a \leq b$ (5mks)

h) Define an inductive set? (2mks)

**QUESTION TWO (20 MARKS)**

a) For any subset E of a metric space $(X, \rho)$, prove that $E^0$ is an open set. (6mks)

a) Consider the metric space $(\Re, d)$ and let $f : \Re \rightarrow \Re$ be defined by $f(x) = |x|$. Show that $f$ is uniformly continuous. (6mks)

c) Show that the limit of a convergent sequence is unique in a metric space (8mks)

## QUESTION THREE (20 MARKS)

a) Show that every infinite set E contains a countable subset A. (7mks)

b) Differentiate between an algebraic and a transcendental number giving examples in each case (3mks)

c) Does the equation $x^2 + 1 = 0$ have a solution in $\Re$? Show your working. (4mks)

d) Define the following terms;

i. A metric space (4mks)

ii. An interior point of a set E (2mks)

## QUESTION FOUR (20 MARKS)

a) Suppose that an open interval $(0,1)$ is equivalent to $\Re$ . Show that $\Re$ is uncountable (10mks)

b) State and provide a proof of Cauchy –Schwarz inequality. (10mks)

## QUESTION FIVE (20 MARKS)

a) Let A and B be nonvoid subsets of $\Re$ and define the set $A + B = \{x + y; x \in A, y \in B\}$,

i. If A and B are bounded above, then show that A+B is also bounded above and sup(A+B)=sup.A+sup.B (5mks)

ii. If A and B are bounded below, then show that A+B is also bounded below and inf.(A+B)=inf.A+inf. B (5mks)

b) For every real numbers $x$ and $a$, $a > 0$, show that $|x| \leq a$ iff $x \in [-a, a]$ (4mks)

c) Let A, B, C be nonvoid sets and $f : A \to B$ and $g : B \to C$ be bijections. Then, prove that $(g \circ f)^{-1}$ exists and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. (6 mks)