(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 203**

COURSE TITLE: VECTOR ANALYSIS

DATE: 18/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1: Compulsory (30 marks)

a. Find the terminal point of the vector $v = 7i - j + 3k$, given that the initial point is $P = (-2, 3, 5)$ (3mks)

b. When do we say two vectors are orthogonal? (2mks)

c. Given the vectors $u = 3i - j + 2k$ and $v = -4i + 2k$ find the angle between u and v (3mks)

d. Show that the diagonals of a parallelogram bisect each other (6 mks)

e. Prove that the dot product of two vectors $u \cdot v = |u||v| \cos\theta$ where θ is the angle between the vectors (4mks)

f. If $f = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$ find
i). curl f ii). div f iii). div(curl f) at (1,-1,1) (5mks)

g. The position of a moving particle is given by
$r(t) = 2\cos ti + 2\sin tj + 3tk$. Find the vectors T, N, B and the curvature (7mks)

Question 2

a. When do we say a vector is irrotational? Prove that that the vector $F = (y + yz)i + (x + 3z^3 + xz)j + (9yz^2 + xy - 1)k$ is a irrotational hence find its potential function ϕ for F (10mks)

b. Evaluate $\int_c xy^2 dx$ on the quarter circle c defined by $x = \cos t, y = 4\sin t$ (10mks)

Question 3

a. Find the work done by $F = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curvature $r(t) = ti + t^2j + t^3k$ from $(0,0,0)$ to $(1,1,1)$ (10mks)

b. State the divergence theorem hence evaluate $\iint_s F.nds$ where the vector $F = 4xzi - y^2j + yzk$ and s is the surface of a cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (10mks)

Question 4

a. Verify Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$ where c is the region bounded by the curves $y = x$ and $y = x^2$ (10mks)

b. Define the following terms (10mks)

i). The unit tangent **T**

ii). The principal normal **N**

iii). The bi-normal **B**

iv). The curvature, k

v). Torsion

Question 5

a. Verify Stokes' theorem for $F = xyi + yzj + xzk$ where s is that part of the cylinder $z = 1 - x^2$ for $0 \leq x \leq 1, -2 \leq y \leq 2$ assuming s is oriented upward (10mks)

b. Let $F = x^2 + y^2 + z^2$; evaluate $\iiint_v Fdv$ where v is the region bounded by $x + y + z = a \quad (a > 0), x = 0, y = 0, z = 0$ (10mks)