(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 203**

COURSE TITLE: VECTOR ANALYSIS

DATE: 08/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1: Compulsory (30 marks)

a. Distinguish between a scalar and a vector (2mks)

b. Show that the addition of any two vectors is commutative (3mks)

c. If $\vec{A} = 4i - 5j + 2k$, $\vec{B} = 6i + 3j - 2k$ and $\vec{C} = 4i + 3j - 5k$ find the scalar product $(\vec{A} \times \vec{B}) \cdot \vec{C}$ (3mks)

d. Find a unit vector perpendicular to both $\vec{A} = i - 2j + k$ and $\vec{B} = 3i + j - 2k$ (3mks)

e. If $\vec{r} = (t^3 + 2t)i - 3e^{-2t}j + 2\sin 5tk$, find
i). $\left|\frac{dr}{dt}\right|$ ii). $\frac{d^2r}{dt^2}$ at $t = 0$.
Give a possible physical significance (5mks)

f. Prove that the dot product of two vectors $\vec{u} \cdot \vec{v} = |u||v|\cos\theta$ where θ is the angle between the vectors (4mks)

g. If $\vec{A} = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$ find
i). curl A
ii). div A (3mks)

h. The position of a moving particle is given by
$r(t) = 2\cos ti + 2\sin tj + 3tk$.
Find the vectors T, N, B and the curvature (7mks)

Question 2

a. If $f = x\sin yz$, find the gradient of f and the directional derivative of f at $(1, 3, 0)$ in the direction of $v = i + 2j - k$ (5mks)

b. Find scalars a, b, c such that the vector $\vec{F}$ is given by

$$\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$$

is irrotational. Hence express $\vec{F}$ as a gradient of a scalar function (8mks)

c. An object starts from rest at the point $P = (1, 2, 0)$ and moves with an acceleration of $a(t) = j + 2k$ where $|a(t)|$ is measured in feet per second. Find the location of the object after $t = 2$ seconds (7mks)

Question 3

a. Define the term 'line integral' of a continuous vector field (3mks)

b. Find the work done by $\vec{F} = (y - x^2)i + (z - y^2)j + (x - z^2)k$ over the curvature $r(t) = ti + t^2j + t^3k$ from $(0,0,0)$ to $(1,1,1)$ (7mks)

c. Let $\vec{F} = x^2 + y^2 + z^2$; evaluate $\iiint_v F dv$ where v is the region bounded by $x + y + z = a \quad (a > 0), x = 0, y = 0, z = 0$ (10mks)

Question 4

a. Evaluate $\int_c A.dr$ where $\vec{A} = 3xyi - y^2j$ along $y = 2x^2$ in the $x - y$ plane from $(0,0)$ to $(1,2)$ (5mks)

b. State the divergence theorem hence evaluate $\iint_s F.nds$ where the vector $F = 4xzi - y^2j + yzk$ and s is the surface of a cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (10mks)

c. State and prove the Frenet Serret formulas (5mks)

Question 5

a. Verify Greens theorem for $\oint_c (xy + y^2)dx + x^2dy$ where c is the region bounded by the curves $y = x$ and $y = x^2$ (8mks)

b. Verify the Stokes' theorem for the vector field $\vec{F} = 2zi + 3xj + 5yk$ taking s to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and the curve to be positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of s in the xy plane (12mks)

END

ALL THE BEST