(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 203**

COURSE TITLE: VECTOR ANALYSIS

DATE: 13/5/2016 **TIME:** 2.00 -4.00 P.M.

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION 1.

(a) Distinguish between a scalar and a vector quantity and give two examples of each. **(4mks)**

(b) Given that **A** = 2i – 3j – k, and **B** = i + 4j – 2k, show that**A**×**B** = -**B** × **A**. **(5mks)**

(c) Find the unit vector parallel to the resultant of the vectors $\mathbf{r_1}$ = 2i + 4j – 5k, $\mathbf{r_2}$ = i + 2j + 3k ; Resultant vector R = $\mathbf{r_1} + \mathbf{r_2}$. **(5mks)**

(d) Evaluate the directional derivative of $\varphi = x^2 yz + 4xz^2$ at (1, -2, -1) in the direction 2i –j – 2k. **(5mks)**

(e) Determine the divergence and curl of the vector field **F** = $x_i + y_j$. **(5mks)**

(f) State the following theorems.

(i)Stokes theorem. **(3mks)**

(ii) Greens theorem. **(3mks)**

QUESTION 2

(a) Find the dot product **A . B** given that **A** = 3i + 3j and **B** = -5i. **(5mks)**

(b) Find θ for non zero vectors **A** and **B** where **A** = 2i – j + 3k and **B** =I + j + k. **(5mks)**

(c) Find a vector perpendicular to the plane determined by the 3 vectors **A** =(1, 3, 2), **B** = (4, -1, 1) and **C** =(3, 0, 2). **(5mks).**

(d) Determine a unit vector perpendicular to the plane of **P** = 2i – 6j- 3k and **Q** = 4i + 3j - k.**(5mks)**

QUESTION 3.

Use Green's Theorem to evaluate the following

(a) $\oint_c \; xydx - x^2y^3\, dy$ where c is the triangle with vertices (0, 0), (0, 1) and (1, 1) positively oriented. **(6mks)**

(b) Show how the Greens theorem may work with regions with holes. **(14mks)**

QUESTION 4

(a) State the divergence theorem. **(3mks)**

(b) Using four points describe the divergence theorem. **(9mks)**

(c) Use the divergence theorem to evaluate $\iint_S \; \boldsymbol{F} \cdot ds$ where **F** =xyi $-\frac{1}{2}y^2$j +zk and the surface consists of 3 surfaces z = 4- $3x^2$ -$3y^2$, $1 \leq z \leq 4$ on the top, $x^2 + y^2 = 1$, $0 \leq z \leq 1$ on the sides and z = 0 on the bottom. **(8mks)**

QUESTION 5

(a) Find the gradient of the scalar field **W** = 10r $\text{Sin}^2\theta\text{Cos}\theta$. **(3mks)**

(b) Given**Q** = x^2y^2 +xyz, compute ∇**W** and the direction derivative $\frac{dW}{dl}$ in the direction $3a_x + 4a_y + 12a_z$ at (2, -1, 0) .**(3mks)**

(c) Find the angle at which the line x= y = 2z intersects the ellipsoid $x^2 + y^2 + 2z^2 = 0$.**(14mks)**