(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF EDUCATION AND BACHELOR OF SCIENCE

COURSE CODE: **MAT 203**

COURSE TITLE: **VECTOR ANALYSIS**

DATE: 18/01/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question 1: Compulsory (30 marks)

a. Show that the addition of any two vectors is commutative (2mks)

b. When do we say two vectors are orthogonal? (2mks)

c. Given $\vec{A} = 5t^2i + tj - t^3k$ and $\vec{B} = \sin ti + \cos tj$ find $\frac{d}{dt}(\vec{A} \cdot \vec{B})$ (3mks)

d. Define a conservative force field hence show that a force $\vec{F}$ defined by

$$\vec{F} = (y^2z^3 - 6xz^2)i + 2xyz^3j + (3xy^2z^2 - 6x^2z)k$$

is conservative (4mks)

e. Find a unit vector perpendicular to both $\vec{A} = i - 2j + k$ and $\vec{B} = 3i + j - 2k$ (3mks)

f. If $\vec{A} = (x^2y^3 - z^4)i + 4x^5y^2zj - y^4z^6k$ find
i). curl A (2mks)
ii). div A (2mks)

g. Find the work done in moving a particle in the force field

$$\vec{F} = 3x^2i + (2xz - y)j + zk$$

along the straight from $(0,0,0)$ to $(2,1,3)$ (5mks)

h. The position of a moving particle is given by
$r(t) = 3\cos ti + 3\sin tj + 4tk$.
Find the vectors T, N, B and the radius of torsion (7mks)

Question 2

a. If $f = x\sin yz$, find the gradient of f and the directional derivative of f at $(1,3,0)$ in the direction of $v = i + 2j - k$ (5mks)

b. Show that $f(x,y,z) = y^2z^3i + 2xyz^3j + 3xy^2z^2k$ is irrotational hence express f as a gradient of a scalar function (8mks)

c. An object starts from rest at the point $P = (1,2,0)$ and moves with an acceleration of $a(t) = j + 2k$ where $|a(t)|$ is measured in feet per second. Find the location of the object after $t = 2$ seconds (7mks)

Question 3

a. Define the term 'line integral' of a continuous vector field (3mks)

b. Find the work done by $\vec{F} = x^2yi - xyj$ over the curvature $r(t) = t^3i + t^4j;\ 0 \leq t \leq 1$ (7mks)

c. Let $\vec{F} = x^2 + y^2 + z^2$; evaluate $\iiint_v Fdv$ where v is the region bounded by $x + y + z = a \quad (a > 0), x = 0, y = 0, z = 0$ (10mks)

Question 4

a. Evaluate $\int_c A.dr$ where $\vec{A} = 3xyi - y^2j$ along $y = 2x^2$ in the $x - y$ plane from $(0, 0)$ to $(1, 2)$ (4mks)

b. State the divergence theorem hence evaluate $\iint_s F.nds$ where the vector $F = 4xzi - y^2j + yzk$ and s is the surface of a cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ (8 mks)

c. Verify Greens theorem for $\oint_c (xy + y^2)dx + x^2dy$ where c is the region bounded by the curves $y = x$ and $y = x^2$ (8mks)

Question 5

a. State and prove the Frenet Serret formulas (8mks)

b. Verify the Stokes' theorem for the vector field $\vec{F} = 2zi + 3xj + 5yk$ taking s to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and the curve to be positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of s in the xy plane (12mks)

END

ALL THE BEST