



(Knowledge for Development)

KIBABII UNIVERSITY

MAIN EXAMINATION

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS AND EDUCATION SCIENCE/ARTS (SB)

COURSE CODE: MAT 203

COURSE TITLE: VECTOR ANALYSIS

DATE: 06/12/18

TIME: 11.30-1.30PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 2 Printed Pages. Please Turn Over.

QUESTION 1.

- (a) Distinguish between a scalar and a vector quantity and give two examples of each. **(4mks)**
- (b) Given that $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$, and $\mathbf{B} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, show that $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$. **(5mks)**
- (c) Find the unit vector parallel to the resultant of the vectors $\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, $\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$;
Resultant vector $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2$. **(5mks)**
- (d) Evaluate the directional derivative of $\phi = x^2 yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. **(5mks)**
- (e) Determine the divergence and curl of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$. **(5mks)**
- (f) State the following theorems.
- (i) Stokes theorem. **(3mks)**
- (ii) Greens theorem. **(3mks)**

QUESTION 2

- (a) Find the dot product $\mathbf{A} \cdot \mathbf{B}$ given that $\mathbf{A} = 3\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = -5\mathbf{i}$. **(5mks)**
- (b) Find θ for non zero vectors \mathbf{A} and \mathbf{B} where $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{B} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. **(5mks)**
- (c) Find a vector perpendicular to the plane determined by the 3 vectors $\mathbf{A} = (1, 3, 2)$, $\mathbf{B} = (4, -1, 1)$ and $\mathbf{C} = (3, 0, 2)$. **(5mks)**.
- (d) Determine a unit vector perpendicular to the plane of $\mathbf{P} = 2\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. **(5mks)**

QUESTION 3.

Use Green's Theorem to evaluate the following

- (a) $\oint_c xy dx - x^2 y^3 dy$ where c is the triangle with vertices $(0, 0)$, $(0, 1)$ and $(1, 1)$ positively oriented.
- (b) Show how the Greens theorem may work with regions with holes. **(20mks)**

QUESTION 4

- (a) State the divergence theorem.
- (b) Using four points describe the divergence theorem.
- (c) Use the divergence theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{s}$ where $\mathbf{F} = x\mathbf{i} - \frac{1}{2}y^2\mathbf{j} + z\mathbf{k}$ and the surface consists of 3 surfaces $z = 4 - 3x^2 - 3y^2$, $1 \leq z \leq 4$ on the top, $x^2 + y^2 = 1$, $0 \leq z \leq 1$ on the sides and $z = 0$ on the bottom. **(20mks)**

QUESTION 5

- (a) Find the gradient of the scalar field $\mathbf{W} = 10r \sin^2\theta \cos\theta$. **(3mks)**
- (b) Given $\mathbf{Q} = x^2 y^2 + xyz$, compute $\nabla \mathbf{W}$ and the direction derivative $\frac{dW}{dl}$ in the direction $3a_x + 4a_y + 12a_z$ at $(2, -1, 0)$. **(3mks)**
- (c) Find the angle at which the line $x = y = 2z$ intersects the ellipsoid $x^2 + y^2 + 2z^2 = 0$. **(14mks)**