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(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 223

COURSE TITLE: DYNAMICS I

DATE: 14/09/17 **TIME:** 3.00 PM-5.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

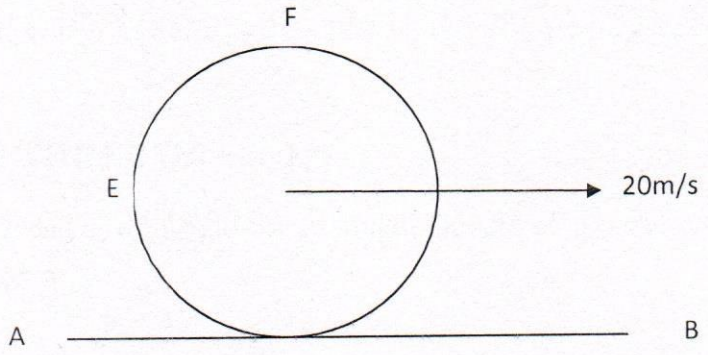
TIME: 2 Hours

This Paper Consists of 4 Printed Pages. Please Turn Over.

QUESTION ONE (30 marks)

a. A rigid body is rotating with angular speed 7 radians per second about a fixed axis through the points $A(2,3,-1), B(-4,0,1)$. The rotation is in the left handed sense relative to $\vec{AB}$. Find the angular velocity vector of the body. (4 marks)

b. A cylinder of radius $1m$ rolls without slipping along a horizontal plane AB as shown in the figure. Its centre has a uniform velocity of $20m/s$. Find the velocity of points D, E and F on the circumference of the cylinder.



(6 marks)

c. Prove that the magnitude of a vector $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ is $|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ (3 marks)

ii. Given $\vec{p} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{q} = -\hat{i} - 4\hat{j} + 3\hat{k}$, find a unit vector perpendicular to both $\vec{p}$ and $\vec{q}$. (3 marks)

d. i. Show that for a body rotating about an axis with constant angular acceleration α, $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ where ω, ω_0, θ and θ_0 are the angular velocity, initial angular velocity, angular displacement and inititial angular displacement respectively. (5 marks)

ii. A flour mill shaft motor is switced on and reaches to the rated speed of $1800rpm$ with an acceleration of $4rad/s^2$. When it is switched off, it decelerates at the rate of $-2rad/s^2$. Determine the number of revolutions it has turned to

1. attain the rated speed (3 marks)
2. come to rest (3 marks)

e. find the angle between the vectors $\vec{r} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{s} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ (3 marks)

QUESTION TWO (20 marks)

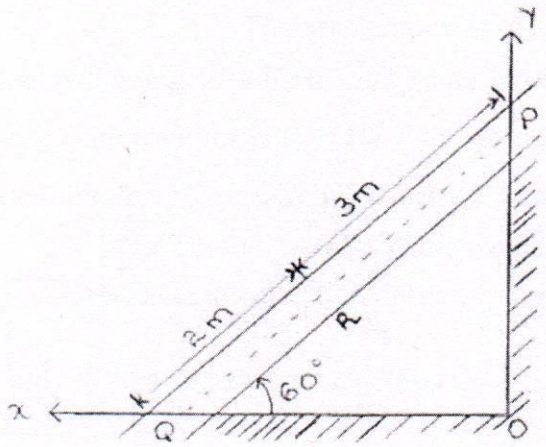
a. i. Derive the formula for the acceleration of a particle in the polar co-ordinate system: $\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$ (9 marks)

ii. A particle sliding along a radical groove in a rotating turntable has polar coordinates at time t given by $r = ct$ and $\theta = \Omega t$ where c and Ω are positive constants. Find the velocity and acceleration vectors of the particle at time t and find the speed of the particle at time t. Deduce that $t > 0$, the angle between the velocity and acceleration vectors is always acute. (8 marks)

b. Given $\vec{r}_1 = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{r}_2 = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\vec{r}_3 = -\hat{i} + 2\hat{j} + 2\hat{k}$ determine $\left|\vec{r}_1 + \vec{r}_2 + \vec{r}_3\right|$ (3 marks)

QUESTION THREE (20 marks)

a) Two ends P and Q of a rigid bar of length 5m slide along the $y-$ and $x-$axes respectively as shown below:



If the velocity of point P is $10m/s$ vertically downwards, determine the

i. Velocity of point Q (5 marks)

ii. Angular velocity of the rigid bar. (3 marks)

iii. Velocity of the point R when it makes an angle of 60^0 with the horizontal. (8 marks)

b) The position of an electron in metres is given by $\vec{r} = 3t\hat{i} - 4t^2\hat{j} + 2\hat{k}$ where t is the time in seconds. Determine the magnitude and direction of the velocity of the electron at $t = 2\,\text{seconds}$. (4 marks)

QUESTION FOUR (20 marks)

a. Given $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{k}$ and $\vec{c} = \hat{i} - 5\hat{j} + 3\hat{k}$, verify that

i. $\vec{a}.(\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}).\vec{c}$ (4 marks)

ii. $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$ (6 marks)

b. Consider a rigid body rotating about a fixed axis AA', show that the acceleration $\vec{a}$ of a point P of the body with position vector $\vec{r}$ is given by $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$ where α is the angular acceleration and ω is the angular velocity. (6 marks)

c. Distinguish between;

i. Rectilinear and curvilinear translation motion (2 marks)

ii. Rotation motion about a fixed axis and general plane motion (2 marks)

QUESTION FIVE (20 marks)

a. The position of a particle P at a time t is given by $\vec{r} = (2t^2 - 3)\hat{i} + (4t + 4)\hat{j} + (t^3 + 2t^2)\hat{k}$. Find

i. The distance OP when $t = 0$ (2 marks)

ii. The velocity when $t = 1$ (2 marks)

iii. The acceleration of P when $t = 2$ (2 marks)

b. A fish swimming in a horizontal plane has a velocity $\vec{v}_0 = 4\hat{i} + \hat{j}$ at a point in the ocean whose position vector is $\vec{r}_0 = 10\hat{i} - 4\hat{j}$ relative to a cliff. After the fish swims with constant acceleration for $20\,\text{sec}\,onds$, its velocity $\vec{v} = 20\hat{i} - 5\hat{j}$

i. What are the x and y components of the aceleration? (2 marks)

ii. What is the direction of the acceleration with respect to the fixed $x - axis$? (2 marks)

iii. Where is fish at $t = 25\,\text{sec}\,onds$, what is its speed and in what direction is it moving? (7 marks)

c. The position of a particle relative to a fixed frame S is $\vec{r} = (t^2 + t)\hat{i} + (t^3 + 2t + 1)\hat{j} + t^4\hat{k}$. If the origin of a rectilinear moving frame S' is moving along a vector $R(t) = (t^3 + 3t + 1)\hat{i} + (t^4 + 2t)\hat{j} + t^5\hat{k}$ relative to S, calculate

i. $\vec{r}'$ (2 marks)

ii. $\vec{v}'$ (1 mark)