(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER

SPECIAL/ SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: MAT 223

COURSE TITLE: DYNAMICS I

DATE: 04/10/18 **TIME:** 3.00 PM-5.00 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

Question One (30mks)

a) A particle moves such that its position vector is given by $\boldsymbol{r} = \cos\omega t\boldsymbol{i} + \sin\omega t\boldsymbol{j}$ where ω is a constant.
   i. Calculate its velocity **v** and acceleration **a** (2mks)
   ii. Show that its velocity **v** is perpendicular to its position vector. (2mks)

b) Calculate the total work done in moving a particle in a force field given by $\boldsymbol{F} = 7xy\boldsymbol{i} - 2z\boldsymbol{j} + 3x\boldsymbol{k}$ along the curve $x = t^2 + 1, y = t^2$, and $z = 2t^2$ from $t = 1$ to $t = 2$.(6mks)

c) Derive the following equation of uniformly accelerated motion $\omega^2 = \omega_0^2 + 2a\theta$(4mks)

d) A pulley of radius 2m is rotating with speed of 400rpm. Calculate its angular velocity and linear velocity.(3mks)

e) Let there be a general direction $\boldsymbol{OM}$around which a vector **OA** of constant magnitude rotates with constant angular velocity ω in a fixed frame. Show that $\frac{dA}{dt} = \boldsymbol{\omega}\times\mathbf{A}$(5mks)

f) A gun is shot horizontally at a target 50m away. The bullet hits the target 1.87cm below the aiming point. (use $g = 9.81ms^{-2}$)
   i. What is the bullet's time of flight? (3mks)
   ii. What is the bullet's muzzle velocity? (2mks)

g) Obtain an expression for velocity in polar coordinates system. (3mks)

Question Two (20mks)

a) A swallow flying in a horizontal plane has velocity $\boldsymbol{v_0} = (\mathbf{5}\boldsymbol{i} + \mathbf{2}\boldsymbol{j})\boldsymbol{ms^{-1}}$ at a point in the sky whose position vector is $\boldsymbol{r}_0 = (10\boldsymbol{i} - 3\boldsymbol{j})m$ relative to a point on the earth's surface. After the swallow flies with constant acceleration for 22 seconds, its velocity is v $= (8i - 6j)ms^{-1}$.
   i. What are the components of acceleration? (5mks)
   ii. Where is it at t=28$_s$ and in what direction is it moving? (5mks)

b) A wheel starting from rest is accelerated at the rate of 5rad/s^2 for an interval of 10s. If it is then made to stop in the next 5s by applying the brakes, find:
   i. The maximum angular velocity attained. (2mks)
   ii. The total angle turned.(4mks)
   iii. Its angular velocity and the total angle turned 2s before stopping. (4mks)

Question Three (20mks)

Show that the expression for acceleration is spherical coordinates is given by

$$a = \left(\ddot{r} - r\dot{\theta}^2\cos^2\theta - r\dot{\emptyset}^2\right)e_r + \left(2\dot{r}\dot{\theta}\cos\emptyset + r\ddot{\theta}\cos\emptyset - 2r\dot{\theta}\dot{\emptyset}\sin\emptyset\right)e_\theta + \left(2\dot{r}\dot{\emptyset} + r\dot{\theta}^2\sin\emptyset\cos\emptyset + r\ddot{\emptyset}\right)e_\emptyset$$

Where the expressions of unit vectors are:

$$\boldsymbol{e}_r = \cos\theta\cos\emptyset i + \sin\theta\cos\emptyset j + \sin\emptyset k$$

$$e_\emptyset = \sin\theta i + \cos\theta j$$

$$e_\theta = -\cos\theta\sin\emptyset i - \sin\theta\sin\emptyset j + \cos\emptyset k$$ (20mks)

Question Four (20mks)

a) A cylindrical roller is in contact at its top and bottom with two conveyor belts. If the top belt runs at a uniform speed of 8m/s and the bottom at 5m/s, find the linear velocity and angular velocity of the roller.(7mks)

b) Find the angular and linear velocities of pulley of radius 2.4m rotating with a speed of 500rmp. (3mks)

c) A 10,000N aircraft is descending on a cylindrical helix. The rate of descent is $z = -10m/s$, the speed is speed is $v = 211m/s$ and $\theta = 0.05\ rad/s$. What is the force in the aircraft and the radius of curvature of the path? (10mks

Question Five (20mks)

consider two frames of reference S and S' with unit vectors $n = (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ and

$n' = (\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ and with a common origin. Let S' rotate with some axis through the origin with angular velocity ω.

Given a particle p whose position vectors are $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $\boldsymbol{r} = x'\boldsymbol{i}' + y'\boldsymbol{j}' + z'\boldsymbol{k}'$ relative to the frames S and S' respectively,

a) show that $\boldsymbol{V} = v' + \omega \, x \, \boldsymbol{r}$, where **v** and **v'** are expressions of velocity vectors in frames S and S' respectively. (10mks)

b) Obtain the expressions for accelerations in both frames and by use of Newton's second Law of motion, obtain the expressions for Coriolis and Centrifugal forces. (10mks)