(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2016/2017 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 222**

COURSE TITLE: **CALCULUS III**

DATE: 29/09/17 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i) Local maximum
   (ii) Local minimum
   (iii) Sequence
   (iv) series (4 mks)

b) Use the 1st principles to determine $\frac{\partial f}{\partial y}$ given that $f(x,y) = x^2 + 2xy^2 + xy$

c) Locate any relative extreme points and determine their nature for the function $f(x_1, x_2, x_3) = x_1 - 4x_1x_2 - x_2{}^2 + 5x_3{}^2 - 2x_2x_3$ (5 mks)

d) A cylinder has a volume of $26.32\ cm^3$.
   (i) What dimensions yields the minimum surface area (5 mks)
   (ii) Find the minimum surface area (1 mk)

e) Let $f(x,y,z) = zcos(xy) - e^{4z^2x} + \ln(xz)$. Find
   (i) f_{xy} (2 mks)
   (ii) f_{xxz} (3 mks)

f) Verify that the Tailor series expansion for the function $f(x) = cosx$ about $x = 0$ is $cosx = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n}}{(2n)!}$ hence find the Maclaurin series for $f(x) = xcosx$ (5 mks)

g) Find the volume in the 1st octant between the planes $z = 0$ and $z = x + y + 2$ and inside the cylinder $x^2 + y^2 = 16$ (5 mks)

QUESTION TWO (20 MARKS)

a) Let $z = x^2e^y$ and $x = t^2 + 3s$ and $y = s^2cost$ find
   (i) $\frac{\partial z}{\partial t}$ (4 mks)
   (ii) $\frac{\partial z}{\partial s}$ (4 mks)

b) If $R = \{x, y \,|\, 0 \leq x \leq 1 \ and \ 1 \leq y \leq 2\}$ evaluate $\iint_R (5y^2 + 3x^2y^2)dA$ (4 mks)

c) Test the convergence of the following limit $s_n = \frac{n^3 - 5}{2n^3 - 3n + 4}$ (3 mks)

d) Find the volume of the solid under the cylinder $x^2 + z^2 = 1$ and above the triangle in the xy-plane bounded by $y = x$, $y = 2x$ and $x = 1$ (5 mks)

QUESTION THREE (20 MARKS)

a) Given that z is a differentiable function near each (x, y) for $2xz^2 - e^{3y}sin5z - 3y^2 = 10$ find

$\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (5 mks)

b) Find the possible local maximum and minimum points of the function $f(x, y) = x^3y - 3xy + 11$ (5 mks)

c) Find the maximum value of production function $f(x, y) = 12x + 3y$ subject to the condition $xy = 4$ (5 mks)

d) Consider the series $1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} + \dots\dots\dots$ using the integral test, determine whether the series converges or diverges (5 mks)

QUESTION FOUR (20 MARKS)

a) Find and classify all critical points of $f(x, y) = x^2 - 12y^2 - 4y^3 + 3y^4$ (7 mks)

b) Find the radius and interval of convergence of the series

$\sum_{n=0}^{\infty} \frac{(-3)^n(x-4)^n}{n6^{n+1}}$ (6 mks)

c) Use the Lagrange multipliers to find the local extrema of the function $f(x, y) = 3x + 4y$ subject to $x^2 + y^2 = 1$ (7 mks)

QUESTION FIVE (20 MARKS)

a) Suppose that $xy + yz^3 + xz = 0$ determines z implicitly as a function of x and y find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (5 mks)

b) Consider the series $\sum_{n=0}^{\infty} \frac{1}{n}$ use ratio theorem to show that the series diverges (5 mks)

c) Determine whether the series converges and if so determine the sum

$\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \dots\dots$ (5 mks)

d) Evaluate $\int_0^1 \int_2^3 \int_{-1}^1 2(xy - yz)dxdydz$ (5 mks)