(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SPECIAL/SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 222**

COURSE TITLE: **CALCULUS III**

DATE: 19/10/18 **TIME:** 11.30 AM -1.30 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Define the following terms
   (i) A critical point
   (ii) Local minimum
   (iii) Sequence
   (iv) Geometric series (4 mks)

b) Use the 1st principles to determine $\frac{\partial f}{\partial y}$ given that $f(x,y) = 2x^2 + 3xy^2 - xy$

c) A cylinder has a volume of $44.78\ cm^3$.
   (i) What dimensions yields the minimum surface area (5 mks)
   (ii) Find the minimum surface area (1 mk)

d) Let $f(x,y,z) = zsin(xy) - e^{2z^2x} + \ln(xz)$. Find
   (i) f_{xx} (2 mks)
   (ii) f_{yxz} (3 mks)

e) Locate any relative extreme points and determine their nature for the function $f(x_1, x_2, x_3) = 25 - {x_1}^2 - {x_2}^2 - {x_3}^2$ (5 mks)

f) Verify that the Tailor series expansion for the function $f(x) = sinx$ about $x = 0$ is $cosx = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$ hence find the Maclaurin series for $f(x) = xsinx$ (5 mks)

g) Find the volume in the 1st octant between the planes $z = 0$ and $z = x + y + 2$ and inside the cylinder $x^2 + y^2 = 16$ (5 mks)

QUESTION TWO (20 MARKS)

a) Given that z is a differentiable function near each (x,y) for $xz^2 + e^{4y}cos5z - 8y^2 = 15$ find
   $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (5 mks)

b) Consider the series $1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} + \cdots\cdots\cdots$ using the integral test, determine whether the series converges or diverges (5 mks)

c) Find the possible local maximum and minimum points of the function $f(x,y) = x^3y - 3xy + 11$ (5 mks)

d) Find the maximum value of production function $f(x,y) = 12x + 3y$ subject to the condition $xy = 4$ (5 mks)

QUESTION THREE (20 MARKS)

a) Suppose that $xy + yz^3 + xz = 0$ determines z implicitly as a function of x and y find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ (5 mks)

b) Consider the series $\sum_{n=0}^{\infty} \frac{1}{n}$ use ratio theorem to show that the series diverges (5 mks)

c) Determine whether the series converges and if so determine the sum $\frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots\ldots$ (5 mks)

d) Evaluate $\int_0^1 \int_2^3 \int_{-1}^1 2(xy + yz)\,dxdydz$ (5 mks)

QUESTION FOUR (20 MARKS)

a) Let $z = y^2 e^x$ and $x = 2t^2 - s$ and $y = s^2 sint$ find

(i) $\frac{\partial z}{\partial t}$ (4 mks)

(ii) $\frac{\partial z}{\partial s}$ (4 mks)

b) If $R = \{x, y \,|\, 1 \leq x \leq 2 \text{ and } 0 \leq y \leq 1\}$ evaluate $\iint_R (8y^2 - 16y^2)dA$ (4 mks)

c) Test the convergence of the following limit $s_n = \frac{n^3 - 5}{2n^3 - 3n + 4}$ (3 mks)

d) Find the volume of the solid under the cylinder $x^2 + z^2 = 1$ and above the triangle in the xy-plane bounded by $y = x$, $y = 2x$ and $x = 1$ (5 mks)

QUESTION FIVE (20 MARKS)

a) Find and classify all critical points of $f(x, y) = 3x^2 - 2xy + y^2 - 3y$ (7 mks)

b) Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-4)^n (x-7)^n}{n 8^{n+1}}$ (6 mks)

c) Use the Lagrange multipliers to find the local extrema of the function $f(x, y) = 3xy$ subject to $2x + y = 8$ (7 mks)