(65)

(Knowledge for Development)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2015/2016 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

MATHEMATICS

COURSE CODE: **MAT 222**

COURSE TITLE: CALCULUS III

DATE: 13/5/16 **TIME:** 2 PM -4 PM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

[Question One, 30mks]

(a) i) Differentiate the following terms giving an example in each case;

Sequence and Series [4mks]

ii) Find a possible n^{th} term for the sequence whose first 5 terms are indicated, hence find the 6^{th} term: $-\frac{1}{5}, \frac{3}{8}, -\frac{5}{11}, \frac{7}{14}, -\frac{9}{17}$ [4mks]

(iii) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

[5mks]

(b) (i) Suppose that $w = f(u, v)$ is a differentiable function and that given $u = ax + by$ and $v = ax - by$. Show that

$$\frac{\partial w}{\partial x} \cdot \frac{\partial w}{\partial y} = ab\left[\left(\frac{\partial w}{\partial u}\right)^2 - \left(\frac{\partial w}{\partial v}\right)^2\right]$$

[6mks]

(ii) If $z = x^2 \tan^{-1} \frac{y}{x}$, find $\frac{\partial^2 z}{\partial x \partial y}$ at $(1, 1)$. [4mks]

(iii) If $z = e^{xy^2}, x = t\cos t, y = t\sin t$, compute $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$. [3mks]

(c) Using the implicit theorem, find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation

$$x^3 + y^3 + z^3 + 6xyz = 1$$

[4mks]

[Question Two, 20mks]

(a) (i) Define Taylor Series expansion of a function $f(x)$ about a point $x_0 = a$. [2mks]

(ii) Verify that the Taylor series expansion for the function $f(x) = \cos x$ about $x = 0$ is;

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Hence find the Maclaurin series for the function $f(x) = x \cos x$ [6mks]

(b) Find the directional derivative of $F = x^2yz^3$ along the curve $x = e^{-u}$, $y = 2\sin u + 1$, $z = u - \cos u$ at the point p where $u = 0$. [7mks]

(c) Compute the gradient and the equation of the tangent plane of an ellipsoid

$$2x^2 + 4y^2 + z^2 - 45 = 0 \quad at \quad (2, -3, -1)$$

[5mks]

[Question Three, 20mks]

(a) A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is a minimum? [7mks]

(b) Locate and classify all critical points for $f(x, y) = 4xy - x^4 - y^4$ [8mks]

(c) If $f(\frac{y}{x})$ is any differentiable function of $\frac{y}{x}$ and

$$u = f\left(\frac{y}{x}\right) + (x^2 + y^2)^{\frac{1}{2}}$$

show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (x^2 + y^2)^{\frac{1}{2}}$$

[5mks]

[Question Four, 20mks]

(a) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2 + 2y^2 + z = 16$, the planes $x = 2$, $y = 2$ and the three co-ordinate planes.

[4mks]

(b) Suppose that $w = \sqrt{x^2 + y^2 + z^2}$, $x = \cos\theta$, $y = \sin\theta$ & $z = \tan\theta$, find $\frac{dw}{d\theta}$ when $\theta = \frac{\pi}{4}$.

[5mks]

(c) Find the local and absolute minimum and maximum value of the function $f(x, y) = 6x + 4y - 7$ on the ellipse $3x^2 + y^2 = 28$ [11mks]

[Question Five, 20mks]

(a) The temperature at the point $P(x, y, z)$ in a solid piece of metal is given by

$$f(x, y, z) = e^{2x+y+3z}$$

degrees. In what direction at point $(0, 0, 0)$ does the temperature increase most rapid.

[5mks]

(b) Find local extrema of $f(x, y) = 3x^2 + y^3$ on the circle $x^2 + y^2 = 9$ [8mks]

(c) Evaluate the triple integral

$$\int\int\int_{\Re}(x^2 + y^2 + z^2)dxdydz$$

given that the region $\Re$ is bounded by $x + y + z = a$, $(a > 0)$, $x = 0, y = 0, z = 0$.

[7mks]