(*Knowledge for Development*)

KIBABII UNIVERSITY

UNIVERSITY EXAMINATIONS

2017/2018 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

MAIN EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE

COURSE CODE: **MAT 222**

COURSE TITLE: **CALCULUS III**

DATE: 10/08/18 **TIME:** 9 AM -11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

QUESTION ONE (30 MARKS)

a) Use the 1st principles to determine $\frac{\partial f}{\partial x}$ given that $f(x,y) = 3xy^2 - 4x^2y + 0.75y^3$ (4 mks)

b) A sphere has a volume of $106.34\ cm^3$

(i) Find the radius of the sphere that will yield minimum surface area (4 mks)

(ii) Find the minimum surface area (2 mk)

c) Given that y is a differentiable function near each (x, z) for $4zy^2 - e^{-3x}sin3y + z^2 = 10$ find

$\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial z}$ (5 mks)

d) Find the radius and interval of convergence of the series

$\sum_{n=1}^{\infty} \frac{(-2)^n(x-6)^n}{n3^{n+1}}$ (5 mks)

e) A particle moves in a circular motion such that its position is given by $x = 2sint$ and $y = cos3t$ for any time t. A force of magnitude $f(x,y) = 0.4x^2 + 4y^2 + xy$ is exerted on the particle at a point (x,y). Find an expression for the rate of change of magnitude of the force exerted by the particle with respect to time when $t = 3$ (5 mks)

f) Verify that the Tailor series expansion for the function $f(x) = \cos x$ about $x = 0$ is $sinx = \sum_{n=0}^{\infty} \frac{(-1)^n(x)^{2n}}{(2n)!}$ hence find the Maclaurin series for

$f(x) = xcosx$ (5 mks)

QUESTION TWO (20 MARKS)

a) Evaluate $\lim_{(x,y)\to(2,2)} \frac{x^2-xy}{\sqrt{y}-\sqrt{x}}$ (4 mks)

b) Let $f(x,y,z) = ycos(xz) - e^{-2x^2y} + 4\ln(xyz)$. Find

(i) f_{yy} (2 mks)

(ii) f_{xyz} (3 mks)

c) Find the maximum value of production function $f(x,y) = 6xy$ subject to budget constraint $4x + y = 10$ (5 mks)

d) Locate any relative extreme points and determine their nature for the function $f(x_1, x_2, x_3) = 10{x_1}^2 + 15{x_2}^2 + 5{x_3}^2 - 60x_1 + 90x_2 - 40x_3 + 100$ (6 mks)

QUESTION THREE (20 MARKS)

a) Find and classify all critical points of $f(x,y) = x^2 - 12y^2 - 4y^3 + 3y^4$ (7 mks)

b) Use the Lagrange multipliers to find the local extrema of the function $f(x,y) = 4y^2 + x^3$ Subject to $x^2 + y^2 = 1$ (6 mks)

c) The area of a triangle is given by $A = \frac{1}{2}acsinB$ when $a = 15cm$, $c = 8cm$ and $C = 30^{\circ}$. Find

(i) The rate of change of A with respect to a when c and B are constant (2 mks)

(ii) The rate of change of A with respect to B when a and c are constant (2 mks)

(iii) The rate of change of c with respect to a when A and B are constant (3 mks)

QUESTION FOUR (20 MARKS)

a) Let $z = e^{3x}tany$ and $x = 2st^2 - 4$ and $y = 3t - 2s$ find

(i) $\frac{\partial z}{\partial t}$ (4 mks)

(ii) $\frac{\partial z}{\partial s}$ (3 mks)

b) If $R = \{x, y \,/\, 0 \leq x \leq 3 \;and\; 0 \leq y \leq 5\}$ evaluate $\iint_R (4xy^2 + 3x^2y^3)dA$ (3 mks)

c) Find the volume of the solid bounded by the graphs of $z = 9 - y^2$, $x + z = 2, x = 0$, and $z = 0$ (5 mks)

d) Consider the series $1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{7}} + \cdots\cdots\cdots$ using the integral test, determine whether the series converges or diverges (5 mks)

QUESTION FIVE (20 MARKS)

a) Consider the series $\sum_{n=0}^{\infty} \frac{1}{n}$ use ratio theorem to show that the series diverges (5 mks)

b) Find the area of the portion of the cylinder $x^2 + z^2 = 25$ lying inside the cylinder $x^2 + y^2 = 25$ (5 mks)

c) For what values does the series converge $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$ (5 mks)

d) Find the volume in the 1st octant between the planes $z = 0$, and $z = x + y - 3$ And inside the $x^2 + y^2 = 4$ (5 mks)