



(Knowledge for Development)

KIBABII UNIVERSITY
UNIVERSITY EXAMINATIONS
2017/2018 ACADEMIC YEAR
SECOND YEAR SECOND SEMESTER
MAIN EXAMINATION

**FOR THE DEGREE OF BACHELOR OF EDUCATION AND
BACHELOR OF SCIENCE**

COURSE CODE: MAT 221

COURSE TITLE: CALCULUS II

DATE: 31/07/18

TIME: 9 AM - 11 AM

INSTRUCTIONS TO CANDIDATES

Answer Question One and Any other TWO Questions

TIME: 2 Hours

This Paper Consists of 3 Printed Pages. Please Turn Over.

Question ONE 30 marks

1. a) Evaluate $\int_0^1 \frac{e^x}{1+e^x} dx$ (5mks)

b) State the mean value theorem (2mks)

c) Given a function $y = f(x)$ is a curve continuous in the interval $[a,b]$, show that the length

(L) of an arc of $f(x)$ is given by $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ (5mks)

d) Evaluate $\int \frac{x^3}{(x-2)(x+3)} dx$ (5mks)

e) Determine the surface area of the solid obtained by rotating $y = \sqrt[3]{x}$ for $1 \leq y \leq 2$ about the y-axis (5mks)

f) Calculate the volume generated when $y = x^2$ and the line $y=4$ is rotated about

(i) X - axis (3mks)

(ii) Y - axis (3mks)

g) State the fundamental theorem of calculus (2mks)

Question Two 20 marks

2 (a) Evaluate $\int \sin^6 x \cos^5 x dx$ (5mks)

(b) Evaluate $\int e^\theta \cos \theta d\theta$ (5mks)

(c) Show that $\int_0^\infty x^2 e^{-x} dx = 2$ (5mks)

(d) Evaluate $\int \frac{x^4 - 5x^2 + 6x + 8}{x^3 - 3x^2} dx$ (5mks)

Question Three 20 marks

3. a) Determine the surface area of the solid obtained by rotating $y = \sqrt{a - x^2}$ about x-axis for $-2 \leq x \leq 2$ (7mks)
- b) Find the length of the arc of the curve $x = 3y^{3/2} - 1$ from $y=0$ to $y=4$ (7mks)
- c) Find the volume generated by rotating about x-axis the area enclosed by the curve $y = x^2 + 1, y = 0, x = 0, x = 1$. (6mks)

Question Four 20 marks

4. a) Show that $\int_1^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{4}$ (5mks)
- b) Show that $\int_1^{\infty} e^{-x} \sin x dx = \frac{1}{2}$ (5mks)
- c) Evaluate $\int_{-\pi/2}^{\pi/2} x \sin x dx$ (5mks)
- d) Evaluate $\int x^2 \sin(x^3) dx$ (5mks)

Question Five 20marks

5. a) Find the derivative of $h(x)$ given $h(x) = \int_2^{x^2} \frac{1}{1+t^2} dt$ (5mks)
- b) Find the area under the function $y = f(x)$ defined by $f(x) = \begin{cases} \frac{1}{2}, & 0 \leq x < \frac{1}{2} \\ x, & \frac{1}{2} \leq x \leq 1 \end{cases}$ (5mks)
- c)(i) Differentiate between convergent and divergent integrals (4mks)
- (ii) Evaluate $\int_{-\infty}^1 \frac{dx}{1+x^2}$ and state whether its convergent or divergent (6mks)